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Non-Destructive Weight Measurement by Using a Vibration Model

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Abstract: A method for weighing fruits without separating them from stem is proposed. The base of stem is fixed and a fruit or a cluster of fruits is forced to vibrate. The approximated vibration model is constructed by the use of Transfer Matrix Method. The natural frequency (w) in this model can be represented as a function of weight elements, and the length and stiffness of branch elements of stem. With this function, only w is possible to measure. However, several small weights whose weights are known are attached to weight elements in various combinations. From these equations, unknown parameters are determined so that the weight of each fruit can be obtained by a non-destructive method.

1. Introduction

Optimum weight and quality control is very important for the production management of fruit's or root's vegetables. But the optimum environment control of above vegetables based on the strict reason may not have been established yet. It is a known fact that a part of nutrients made in leaves are distributed to fruits or roots, and a partition ratio of nutrients can be controlled by an environment adjustment.

So a non-destructive and continuous measurement method of living fruit's or root's weight is very useful for the development of optimum environment control. For the measurement of partial weight of the object, it is usually necessary to separate the object to be measured from the main part of the object. In this paper, the measurement of fruit weights by using the vibration model without separating them from the stem is discussed.

2. Basic principle

In this method, first, the base of the stem being fixed, the natural frequency of vibration is measured by the forced vibration of the fruit and its spectral analysis. Next, some small weights whose weights are known, are attached to fruits and the changes of natural frequency are measured. The measured values are given to the fruit's vibration model and fruit weight can be known by the elimination of unknown parameters, that is, each length and each stiffness of stem or stem branches, from the equations of the model.

In the construction methods of the vibration model, there is the analytical method, finite element method, transfer matrix method, and so on. In this paper the transfer matrix method is selected[1]. In this method a fruit or fruit

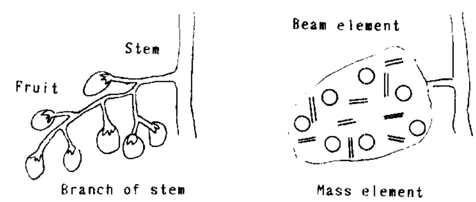

Fig.1 Elements of a fruit or a cluster of fruits

cluster including a stem or branches of stem is divided into weight elements and beam elements as shown in Fig.1.

If $X(x1,x2,\cdots,xl)$, $Y(y1,y2,\cdots,ym)$, $Z(z1,z2,\cdots,zn)$ are expressed weight of weight elements, the length of each beam element, the stiffness of each beam element, respectively, the vibration model can be represented by the following equation.

$$f(w1,X,Y,Z)=0 \qquad (1)$$

where w1 is the natural frequency. In this equation, only w1 is possible to measure, but all other parameters are unknown. However, n+m+l small weights whose weights a1, a2, ---, al+m+n-1 is known, can be attached to each weight element in various combinations. So the following l+m+n independent equations are obtained.

$$f(w2,\ X+a1,Y,Z)=0$$

$$\text{-------} \qquad (2)$$

$$f(wl+m+n-1,X+al+m+n-1,Y,Z)=0$$

From these equations, l+m+n parameters are determined, so that the weight of each fruit weight can be obtained by a non-destructive method. But the solution problem of these equations is thought to be ill-posed. So it is feared that noises in the measurement or errors in the calculation prevent a true solution. In overcoming this problem, it may be effective to adopt some a priori values in x, l, y, m, z, and n.

3. The construction of the vibration model

The state of vibration is classified to the next 3 types in the corresepondence to the kind of fruit's vegetables.

1)Flexural vibration of a fruit
---- green peppers etc.
2)Torsional vibration of a fruit

----- cucumbers etc.

3)Flexural vibration of a cluster of fruits

----- tomato, strawberry, etc

Now let us proceed to the construction of vibration models of the above 3 vibration types.

3.1 Type 1

If stem length is over the value multiplied radius of the fruit by 10 or 20, a beam at a tip of which a multimass is attached can be adopted as this vibration model[2] . In this case from equation (1) and (2) the following equation for the weight of fruit can be obtained.

$$M = \frac{a}{\left(\frac{w1}{w2}\right)^2 - 1} \qquad (3)$$

If stem length is not filled with the above condition, a fruit can not be treated as a multimass. In the transfer matrix method, a fruit can be represented with a model in which some different masses are connected one dimensionally to each other with the massless beams as shown in Fig.2. Natural frequency equation (1) is easily obtained from the production of mass matrix of fruit with transfer matrix of stem, and the two boundary conditions.

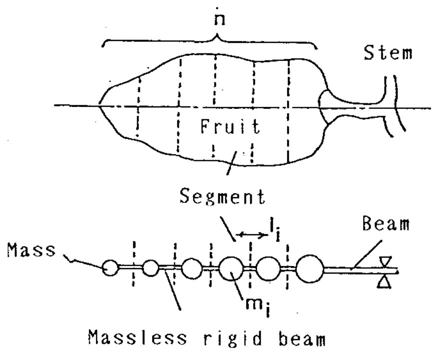

Fig.2 One dimensional multimass model of fruit

The concrete explanation of the method for natural frequency equation is the following. The motion of a mass or beam in the segment No.i is represented by the shear force Vi, the bending moment Mi, the deflection wi, and the slope ψ as shown in Fig.3. EJ in Fig.3 is flexural rigidity.These parameters are represented by the following state vector Zi.

$$Zi = \begin{pmatrix} -w \\ \psi \\ M \\ V \end{pmatrix}_i \qquad (4)$$

Zi and Zi+1 are connected by the transfer matrix Ti.

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The transfer matrices of mass segment Tmi, rigid beam segment Tbi,stem segment Ts are represented respectively by the following ;

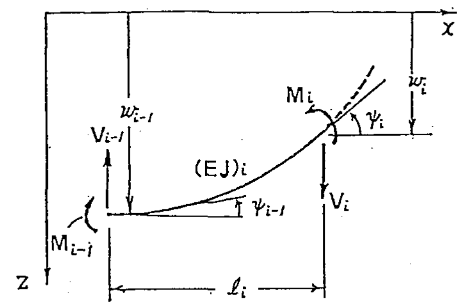

Fig.3 End forces and deflections for massless beam

$$Tmi = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ miw^2 & 0 & 0 & 1 \end{vmatrix} \qquad (6)$$

$$Tbi = \begin{vmatrix} 1 & li & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & li \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (7)$$

$$Ts = \begin{vmatrix} 1 & s & s^2/2EJ & s^3/6EJ \\ 0 & 1 & s/EJ & s^2/2EJ \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (8)$$

The transfer matrix of the total system is

$$U=TM1*TB1*TM2*----*TBn*Ts \qquad (9)$$

and the natural frequency can be obtained from the following equation.

$$\begin{vmatrix} U33 & U34 \\ U43 & U44 \end{vmatrix} = 0 \qquad (10)$$

This equation corresponds to equation (1).

3.2 Type 2

A fruit of this type can be represented by a model in which a fruit is constructed of concentrated masses connected one dimensionally to each other as shown in Fig.4. The moment of inertia I is

$$I = \sum_{i=1}^{N} Mili \qquad (11)$$

where li is the distance between fulcrum and concentrated mass Mi. Natural frequency w of physical pendulum in a small vibration is

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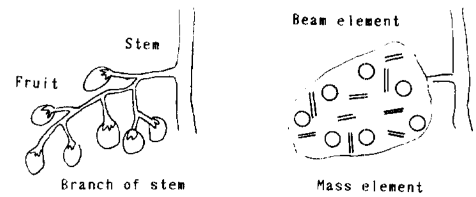

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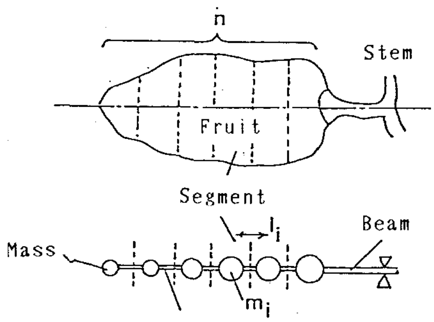

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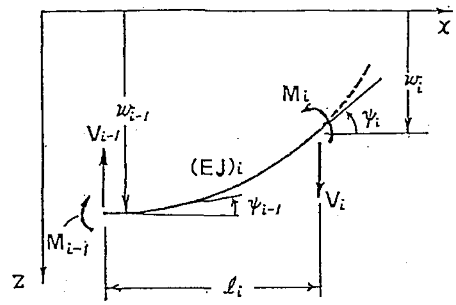

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where M is the mass of fruit, g is acceleration of gravity, h is the distance between fulcrum and centre of gravity, k is the torsional constant of the stem . h can be obtained from the following equation.

$$h=\sum_{i=1}^{N}(M_i l_i)/M \qquad (13)$$

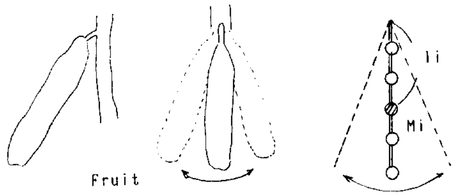

Fig.4 A model of type 2 fruit

3.3 Type 3

In this type the measurement of an individual fruit weight in the fruit clusters is discussed. If an aparatus for the vibration of individual fruit can be set in the cluster, type 1 can be applied. But in type 3 all fruits in a cluster can not but be simultaneously vibrated.

For the simplicity of calculation, the individual fruit is treated as concentrated mass. The longest member of stem which connects to the base of the stem can be treated as a main member. It is considered that branch members of stem spread radially from a main member and a fruit attaches to the tip of the individual branch member.

The stem branches in various directions in 3 dimensional space. If each branch member is considered as a straight line beam, its force or deflection can be described on the standard coordinate system by coordinate conversion. Therefore, the basic model of branch members can be shown as in Fig 5. A model of any branches of stem can be constructed from the extension of this basic model.

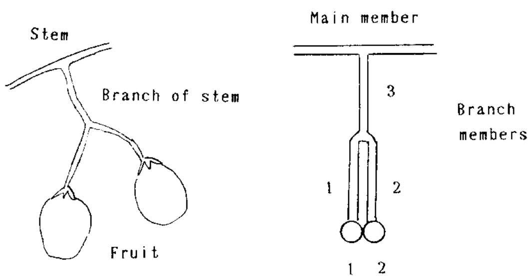

Fig.5 A basic model of branch member

In this basic model branch member 3 is spread vertically downward from the main member . From branch member 3 both branch member 1 and 2 spread further vertically downward. Mass 1, and 2 attaches to the tip of both branch members.

The difference between type 1 and type 3 is in the point that in type 3 the force N with the direction of member's axis acts to the branch member. So the status vector of the beam with length s, Zi, and the transfer matrix Ui of beam are extended to the following equations.

$$Z_i = \begin{bmatrix} u \\ w \\ \psi \\ M \\ N \\ Q \end{bmatrix}_i \qquad (14)$$

$$U_i = \begin{vmatrix} 1 & 0 & 0 & 0 & -s/EF & 0 \\ 0 & 1 & -s & s^2/2EJ & 0 & s^3/6EJ \\ 0 & 0 & 1 & -s/EJ & 0 & -s^2/2EJ \\ 0 & 0 & 0 & 1 & 0 & s \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \qquad (15)$$

where u is the displacement incured by N and EF is tensile rigidity.

Transfer matrices of branch member 1,2,3 and mass m1,m2 are connected into that of main member in the following manner. First, from the following condition at the lower end of the branch members

$$Q_i=0, \quad M_i=o, \quad N_i=m_i w^2 w_i \quad (i=1,2) \qquad (16)$$

and transfer matrix Ui of branch member 1,2, the relation between the force and displacement at the upper end of the branch members are obtained by the following.

$$\begin{bmatrix} M \\ N \\ Q \end{bmatrix}_i = V_i \cdot \begin{bmatrix} u \\ w \\ \psi \end{bmatrix}_i \qquad (i=1,2) \quad (17)$$

where Vi= is the 3*3 matrix.

Next, from the connection condition between branch members 1,2 and 3, the relation between force and displacement at the lower end of branch member 3 is obtained. Now by consideration of the connection conditions of the force and displacement at the connection point, branch member 1,2 and 3, mass 1, 2 are represented as the transfer matrix of main member.

By the extention of basic type of stem's branch shown in Fig.5 , it is possible to constitute a model which can be applied to any type of fruit cluster. Though it is assumed that a fruit at the tip of a branch member is a concentrated mass, each fruit can also be represented as some one dimensionally connected mass elements.

4. Experimental Result

The feasibility of the proposed method is confirmed by an experiment. Experimental sample is a green pepper, whose external shape is drawn in Fig.6. Position Pa is the location where a small metal weight is attached and position Pb is the location where a small metal plate for the detection of vibration is attached. Each plate is fixed to the green pepper by a small sheet of adhesive tape. A fruit is vibrated by an impact generated by the electric coil which is laid under the fruit as shown in Fig.7. A small metal plate for the vibration is also attached at the position Pc by a small sheet of adhesive tape. Natural frequency is obtained by the spectral analysis of vibration. Relation between an attached weight and natural frequency is shown in Table 1.

In this experiment, the following a priori information about segment number and its weight ratio based on the outer shape is adopted. This

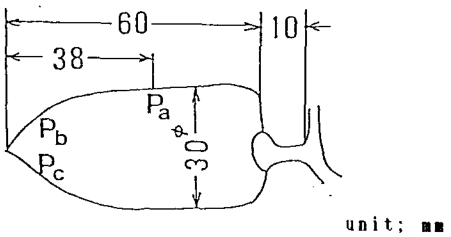

Fig.6 External shape of experimental sample

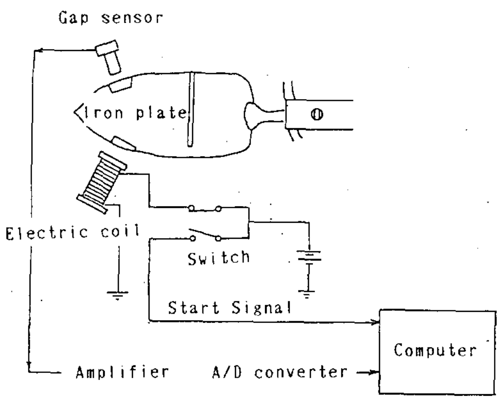

Fig.7 Block diagram of the measurement system

Table 1 Relation between an attached weight and natural frequency

attached weight(g)	natural frequency(rad/sec)
0	51.35
2	49.89
4	47.85

information is summarized in Table 2. Natural frequency equation f which is shown in (1), is constructed from the production of each segment's matrix. So it is difficult to obtain analytically M,EJ,and s. Therefore solution curves for the natural frequency equation are drawn in a s-EJ space. it is expected that at the true M there should be no difference between these curves whether a weight is attached or not.

Solution curves assumed that the weight of fruit is 16g , 17g respectively as shown in Fig.8 and 9. It is estimated that by this method , the weight of green pepper is 17g. After experiment, by separating the sample from the stem , green pepper weight is proved to be 16.5 g . From this result, an example for the validity of the proposed method can be obtained.

Table 2 An assumed segment weight ratio

← Tip of fruit					Base of stem →
0.05	0.14	0.16	0.19	0.22	0.24

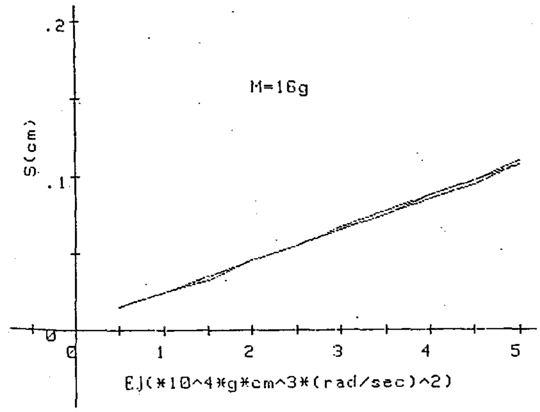

Fig.8 Solution curve on the assumption of M=16 g

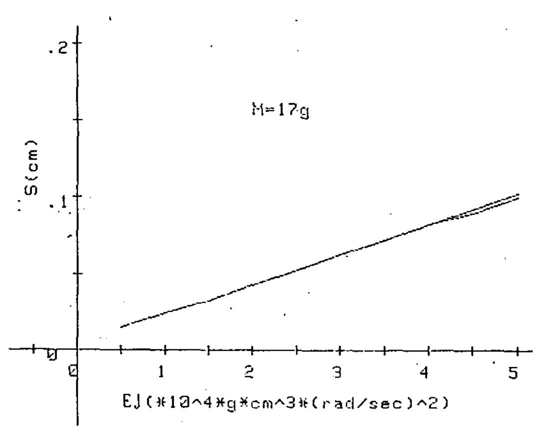

Fig.9 Solution curve on the assumption of M=17 g

Conclusion

A non-destructive weight measurement for living fruit by using a vibration model has been discussed. A construction method of the vibration model is shown and the feasibility of the proposed method is confirmed by experiment. This measurement problem is an ill-posed inverse problem. In this paper, for obtaining a stable solution,some a priori information about segment number and weight ratio of each segment is adopted. But the optimum selection method of a priori information is a future problem.

6. Acknowledgement

I would like to thank prof. H.Yamasaki of The Tokyo University and Dr. M.Takatsuji of Hitachi Ltd. for their continuous encouragement during this research.

7.References

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