'88 KACC 1988. 10. 21~22

A Heuristic Search on Noninferior Solutions to the Halkin-typed Linear Quantized Optimal Control Problem with Two Performance Functions

Tsunehiro MUNAKATA
Hiroshima-Denki Institute of Technology
20-1, Nakano 6-chome, Aki-ku, Hiroshima-city
JAPAN

Abstract

In quantized control systems, the control values can take only given discrete (e.g. integer) values.

In case of dealing with the control problem on the discrete-time, final-stage fixed, quantized control systems with multidimensional performance functions, the first thing, new definition on noninferior solutions in these systems is necessary because of their discreteness in state variables, and the efficient search for those solutions at final-stage is unavoidable for seeking their discrete-time optimal controls to these systems.

In this paper, to the quantized control problem given by the formulation of Halkin-typed linear control systems with two performance functions, a new definition on noninferior solutions of this system control problem and a heuristic effective search on these noninferior solutions are stated.

By use of these concepts, two definitions on noninferior solutions and the algorithm consisted of 8 steps and attained by geometric approaches are given. And a numerical example using the present algorithm is shown.

1. Introduction

Control systems whose control can take only discrete values are called quantized systems.[1)2)] Such systems have usually been optimized by mathematical programming methods.[3)] In comparison of quantized control systems with nomal continuous control systems, it can be stated that the accuracy is less, but reverse control is simplified and fixed anticipating acquisition control can be realized. These points are considered essential for the problem of switching systems, ecological systems handling a number of individuals, investments, traffic control, etc..[4)]

In this paper, new definitions on noninferior discrete solutions and a heuristic search to their solutions at final-stage k are given. Where, these are unavoidable, substantial affairs in case of the optimal control to the discrete-time, final-stage-fixed, quantized systems with multidimensional performance functions.[5)]

Now, I will explore linear discrete-time variant quantized systems with two performance functions and investigate noninferior discrete solutions at final-stage k with a heuristic search, which are needed to get all discrete-time controls of this control problem.

Finally, the algorithm with 8 steps based on this method is described and compared with another method and a numerical example is shown.

2. A Heuristic Search on Noninferior Solutions to this Quantized Control Problem

2.1 Formulation of this quantized control problem[5)]

The state equation is:

$$x(i+1)-x(i)= A(i)x(i)+ g(i,u(i)) \quad \ldots(1)$$
$$i= 0,1,\ldots,k-1$$

where,

x is the state vector, element of the (n+1)-dimensional real space R^{n+1}

u is the control vector, element of subspace Ω (Ω is admissible discrete controls' set which is consisted of bounded, finite elements) of the r-dimensional real space R^r

A is a (n+1)-dimensional square matrix

g is a (n+1)- dimensional vector function

k is some fixed positive integer value.

It is assumed that

(i) g(i,u) is bounded for u in each i= 0,1,...,k-1

(ii) I+ A(i) is nonsingular matrix in each i= 0,1,...,k-1.

Here, I is a unit matrix.

Initial condition is the next:

$x(0) = x_0$.

Then this control problem is given as follow.

Determine the sequence of controls $\{u(i)\}$ $i = 0,1,\ldots,k-1$ and the corresponding state values $\{x(i)\}$ $i = 0,1,\ldots,k$ such that for given initial value x_0, satisfying state equation (1) and

$x(k) \in R^{n+1}$ (terminal condition is free) ...(2)

and two performance functions:

$$\begin{bmatrix} x^n(k) \\ x^{n+1}(k) \end{bmatrix} \text{ is maximum.} \quad \ldots(3)$$

Now, $x^n(k)$, $x^{n+1}(k)$ are denoted the n-th, (n+1)-th element of x(k), respectively. Where, this formulation is derived from the one of Halkin's continuous valued control problem.[6)] See Fig. 1 .

2.2 Definitions of multidimensional solutions

Definitions of multidimensional solutions on continuous values are well-known, for example, in reference 7).

In this control problem, the variables used in this are discrete, therefore new definitions on them are necessary.[5)]

Σ is state space R^{n+1}

W is reachable vector's subspace of Σ .

When some $x \in R^{n+1}$ is decided, the following notations are given.

$\Sigma_s(x)$ is subclass of Σ which are larger (superior to) than the values of x in performance functions

$\Sigma_{IE}(x)$ is subclass of Σ which are inferior to or equal to x

$\Sigma_{\sim}(x)$ is subclass of Σ which are not comparable with x.

Now, these three subclasses are disjoint and associating of Σ .

The next two new definitions are provided:

Definition 1

If the relation $W \subset \Sigma_{IE}(x)$ holds, $x \in W$ is called complete optimal.

Definition 2

If the relation $\Sigma_s(x) \cap W = \phi$ holds, $x \in W$ is called noninferior.

Now, let x is convex hull points of W, and ϕ is null set. Here, convex hull points x of W are said that x is boundary points of co W (convex hull of W), where W is bounded discrete valued reachable set in which contained x. See Fig. 2 and Fig. 3 .

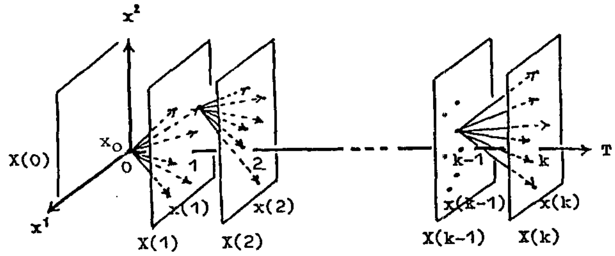

Fig. 1 A transition of state vectors in each stage based on state equations

2.3 An algorithm on a heuristic search of noninferior solutions

In this control problem, as given in 2.1, the degree of dimension of state vector is n+1, and the number of performance functions is two. Therefore, the degree of dimension on Movement space is n-1 and it is (n-1)-dimensional space spanned by $e^1, e^2, \ldots, e^{n-1}$; now, e^i is a unit vector that i-th element is 1.

And the degree of space (plane) on performance functions are two. So I will research the noninferior solutions to the results of projections of (n+1)-dimensional state values x on the 2-dimensional performance plane.

To this control problem, a way of control by the theorem of Quantized Discrete Maximum Principle is known.[5)] The theorem is the next.

Quantized Discrete Maximum Principle

If the sequences $\{\hat{u}(i)\}$ $i = 0,1,\ldots,k-1$ and corresponding $\{\hat{x}(i)\}$ $i = 0,1,\ldots,k$ are optimal, then there exists a sequence of nonzero vectors $\{\hat{p}(i)\}$ $i = 0,1,\ldots,k$ such that

1) Maximization of Hamiltonian

$$\langle A(i)\hat{x}(i) + g(i,\hat{u}(i)),\ \hat{p}(i+1) \rangle \geq \langle A(i)\hat{x}(i) + g(i,u),\ \hat{p}(i+1) \rangle \quad \ldots(4)$$

for all $i = 0,1,\dots,k-1$ and all $u \in \Omega$

2) Adjoint Equations

$$\hat{p}(i) - \hat{p}(i+1) = A(i)^T \hat{p}(i+1) \quad \dots(5)$$

for all $i = 0,1,\dots,k-1$

3) Transversality Conditions

$$\hat{p}(k) = \psi, \ \psi^n \geqq 0, \ \psi^{n+1} \geqq 0. \quad \dots(6)$$

Now, $\langle a, b \rangle$ is denoted inner product of a and b. T is the transpose.

According to this theorem, ψ is nonzero vector whose i-th, (i+1)-th elements are nonnegative, that is

$$\psi = \lambda e^n + (1-\lambda) e^{n+1}$$
$$(0 \leqq \lambda \leqq 1 : \text{constant})$$

:convex combinational (n+1)-dimensional vector.

Where, it is the meaning of "optimal" in this paper that with state equation (1), controls $\{\hat{u}(i)\}$ $i = 0,1,\dots,k-1$ and corresponding $\{\hat{x}(i)\}$ $i = 0,1,\dots,k$ can let the condition be satisfied that $\hat{x}(k)$ is complete optimal or a noninferior solution at final-stage k.

To solve this control problem with Q.D.M.P. (Quantized Discrete Maximum Principle), there are two points:

one is to decide $\hat{p}(k)$ with the condition of $\hat{x}(k)$,

and the other is to decide $\{\hat{u}(k)\}$ $i = 0,1,\dots,k-1$ by Hamiltonian's maximization.

It is the purpose in this paper that I can get the value and condition of $\hat{x}(k)$ for the sake of the decision of $\hat{p}(k)$.

In this system, the reachable sete W(i) at stage i are contained with finite discrete valued state vectors.

$$W(0) = \{x_0\} \quad \dots(7)$$

$$W(i) = \{x(i) : x(i) = [I + A(i-1)]x(i-1) + g(i-1,u(i-1)) \mid u(i-1) \in \Omega\} \quad \dots(8)$$

$$i = 1,2,\dots,k$$

The set of W(k) at final-stage is consisted of finite, bounded, discrete-valued state vectors (points).

Complete optimal or noninferior solution $\hat{x}(k)$ can be gotten by searching from maximum points in e^n-axies to the same points in e^{n+1}-axies about the results of projections on e^n and e^{n+1} performance plane of the set co W(k): convex hull of W(k).

The algorithm follows.

Algorithm

Step 1 Compute x(k) with state equation:

$$x(i+1) = [I + A(i)]\, x(i) + g(i,u(i))$$
$$i = 0,1,\dots,k-1 \text{ and } x(0) = x_0, \ u(i) \in \Omega$$

Step 2 Find $\max\limits_{x(k) \in W(k)} x^n(k)$.

Choose x(k) such that $x^{n+1}(k)$ is maximum, if there are plural maximum.

Step 3 Find $\max\limits_{x(k) \in W(k)} x^{n+1}(k)$.

Choose x(k) such that $x^n(k)$ is maximum, if there are plural maximum.

Step 4 If the result x(k) between Step 2 and Step 3 is the same, go to Step 5. If not, go to Step 6.

Step 5 x(k) is complete optimal solution. It is finished.

Step 6 Let $\hat{x}_1(k)$ be $\max\limits_{x(k)} x^n(k)$.

Let $\hat{x}_2(k)$ be $\max\limits_{x(k)} x^{n+1}(k)$.

The straight line passing on two points $\hat{x}_1(k)$ and $\hat{x}_2(k)$ is drawn. Search the points x(k) in the region which is upper than this line and e^n, e^{n+1}-axies.

If it is not existed x(k), go to Step 7.
If it is existed x(k), go to Step 8.

Step 7 $\hat{x}_1(k)$ and $\hat{x}_2(k)$ (only these two points) are noninferior solutions, and it is finished.

Step 8 Search the points x(k) from $\hat{x}_1(k)$ to $\hat{x}_2(k)$, with the information of angle

$$\max_{x(k)} \left[-\frac{x^{n+1}(k) - x_1^{n+1}(k)}{x^n(k) - x_1^n(k)} \right]$$

and cut the region which searching has been done. These x(k) are noninferior solutions, and it is finished. See Fig. 4 and Fig. 5 .

2.4 Numerical example

Control problem with 5-dimensional state equation (the degree of dimension of moving equation is 3, and performance is 2) is given.

Results on noninferior solutions to this problem by the algorithm in this paper are shown.

Microcomputer SORD M23 is used for the computation.

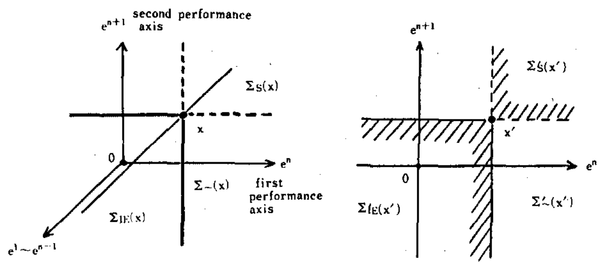

Fig. 2 Spaces $\sum_s(x), \sum_{IE}(x), \sum_{\sim}(x)$ and their projections $\sum'_s(x'), \sum'_{IE}(x'), \sum'_{\sim}(x')$ on the plane spanned by performance axes e^n, e^{n+1}

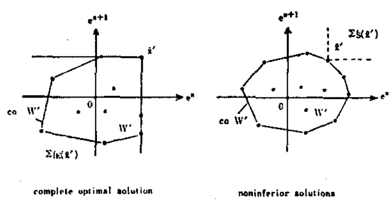

Fig. 3 A complete optimal solution, noninferior solutions in this paper, where co W' is the convex hull of W'

State equations:

$$\begin{bmatrix}x^1(i+1)\\x^2(i+1)\\x^3(i+1)\\x^4(i+1)\\x^5(i+1)\end{bmatrix} - \begin{bmatrix}x^1(i)\\x^2(i)\\x^3(i)\\x^4(i)\\x^5(i)\end{bmatrix} = \begin{bmatrix}2&0&-i^2&1&3\\-3&i&1&0&2\\0&2&-2i&0&1\\i^2&1&0&-2&2\\0&-1&i&3i&1\end{bmatrix}\cdot\begin{bmatrix}x^1(i)\\x^2(i)\\x^3(i)\\x^4(i)\\x^5(i)\end{bmatrix} + \begin{bmatrix}u^1(i)&0&u^3(i)\\0&2i&0\\i&0&-3\\u^1(i)&u^2(i)&1\\1&-2&u^3(i)\end{bmatrix}\cdot\begin{bmatrix}u^1(i)\\u^2(i)\\u^3(i)\end{bmatrix}$$

$i = 0,1,\ldots,4$ ... (9)

Admissible discrete controls:

$$u(i) = \begin{bmatrix}u^1(i)\\u^2(i)\\u^3(i)\end{bmatrix} \in \Omega = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$$

Initial value:

$$x(0) = \begin{bmatrix}x^1(0)\\x^2(0)\\x^3(0)\\x^4(0)\\x^5(0)\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

Performance functions:

$$\begin{bmatrix}x^4(5)\\x^5(5)\end{bmatrix} : \text{maximize}.$$

(the condition on terminal is free)

Result.

Assumption (i), (ii) are satisfied.

To the set W(5) (the number of points of W(5) is 243 (= 3^5)), I will get

$$\hat{x}_1(5) = \begin{bmatrix}7374\\5036\end{bmatrix}, \quad \hat{x}_2(5) = \begin{bmatrix}3692\\6554\end{bmatrix}.$$

I will get a noninferior solution,

$$\hat{x}(5) = \begin{bmatrix}5822\\5036\end{bmatrix}.$$

Thus, I can get the following 6 noninferior solutions,

$$\begin{bmatrix}1323\\-1227\\-533\\7374\\3784\end{bmatrix}\begin{bmatrix}1323\\-1227\\-540\\7374\\3784\end{bmatrix}\begin{bmatrix}2479\\-1343\\-113\\5822\\5036\end{bmatrix}\begin{bmatrix}2479\\-1343\\-120\\5822\\5036\end{bmatrix}\begin{bmatrix}3115\\-635\\391\\3692\\6554\end{bmatrix}\begin{bmatrix}3115\\-635\\384\\3692\\6554\end{bmatrix}.$$

3. Conclusion

For unavoidable search on noninferior solutions of the control problem given in 2.1, a heuristic search (especially the usage of line between $\hat{x}_1(k)$ and $\hat{x}_2(k)$) based on the geometric approach on the performance plane and algorithm, numerical example are stated.

In comparison between this method and the method in reference 8), the number of searching points is less two third (2/3), concretely 42 points to 12 points. Therefore, the efficiency of searching is rising.

Acknowledgement

The author would like to appreciate Dr. Hikaru YAMADA, Associate Professor of Hiroshima-Denki Institute of Technology for his supports and valuable comments on the study.

References

1) Miyamichi and Fukao: Controllability of quantized control systems (in Japanese), Transactions of SICE, JAPAN, Vol. 12, No. 2, 1976
2) Y. Anzai: A note on reachability of discrete-time quantized control systems, IEEE transactions of Automatic Control, AC-19, No. 5, 1974
3) Havira and Lewis: Computation of quantized controls using differential dunamic programming, IEEE transactions of Automatic Control, AC-17, No. 2, 1972
4) T. Munakata: A study on the discrete quantized optimal control systems and its applications (in Japanese), Ph. D. Dissertation, Keio University 1977
5) T. Munakata: An optimal quantized solution to the Halkin-typed discrete control problem with two performance functions (in Japanese), Systems and control, JAPAN, Vol. 28, No. 11, 1984.
6) H. Halkin: Optimal control for systems described by difference equations, Advances in control systems; theory and applications, Vol. 1, 1964
7) K. Shimizu: Systems optimization theory (in Japanese), Corona publishing co. 1976
8) T. Munakata: On noninferior solutions for the Halkin-type quantized linear control problem with a two dimensional vector-valued performance function (in Japanese), Electronics and communications in Japan, Vol. J-67 A, No. 9, 1984.

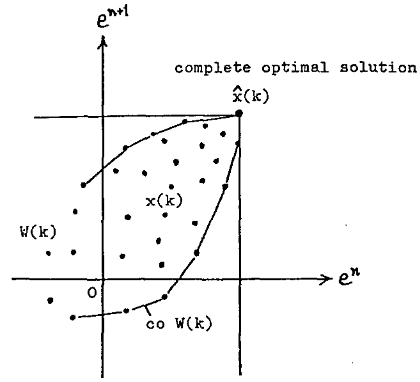

Fig. 4 A complete optimal solution $\hat{x}(k)$ in step 5 of this algorithm in the paper

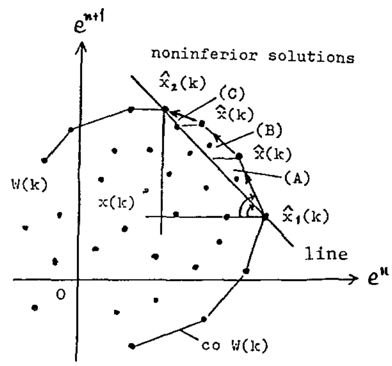

Fig. 5 Noninferior solutions in step 8 of this algorithm in the paper