# ADAPTIVE CONTROLLER WITH FAST CONVERGENCE

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**Abstract**: A way of improving the transient performance is suggested for a class of model reference adaptive control systems. To increase the convergence rate of a model following error, an error feedback term is incorporated into the control law.

## Introduction

Narendra and Valavani (1) presented the design of stable adaptive controllers based on a model reference approach and a Lyapunov stability method for unknown plant with single input and single output. This paper is concerned with improving the transient performance of an adaptive controller designed along the lines of Narendra and Valavani. To increase the convergence rate of a model following error, an error feedback term is incorporated into the control law.

## Formulation of the State Equation

Consider the transfer function of an unknown plant given by

$$\alpha(s)y(s) = k_p\beta(s)u(s) \qquad (1)$$

where

$$\alpha(s) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n \qquad (1.a)$$

$$\beta(s) = s^{n-1} + \beta_2 s^{n-2} + \cdots + \beta_n \qquad (1.b)$$

Also, u(s) and y(s) are the Laplace transforms of scalar input u(t) and scalar output y(t), $\alpha(s)$ and $\beta(s)$ are relatively prime with $\beta(s)$ a Hurwitz polynomial, and $k_p$ is a constant gain. It is further assumed that only n and the sign of $k_p$ are known for use in the design of the controller. Without loss of generality, the sign of $k_p$ is assumed to be positive

Let us choose a monic Hurwitz polynomial of degree (n-1) denoted by f(s)

$$f(s)=s^{n-1}+f_2 s^{n-2}+\cdots+f_n \triangleq s^{n-1}+f^T d(s) \qquad (2)$$

where $f^T=(f_2, f_3, \cdots, f_n)$ and $d^T(s)=(s^{n-2}, \cdots, s, 1)$. Let us introduce auxiliary vector signals v and w generated by

$$v= \frac{d(s)}{f(s)}\, y, \quad w= \frac{d(s)}{f(s)}\, u \qquad (3)$$

Note that the filters in (3) are actually implemented as

$$\left.\begin{aligned} \dot{v} &= Fv + hy \\ \dot{w} &= Fw + hu \end{aligned}\right\} \qquad (4)$$

where the (n-1) x (n-1) matrix F and the (n-1)-dimensional vector h are the companion-form realization given by

$$F = \left[\begin{array}{c|c} \multicolumn{2}{c}{f^T} \\ \hline 0 & I \end{array}\right], \quad h = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (4.a)$$

Dividing (1) by f(s) in (2) and specifying the state vector as $x(t)=(y(t), v(t)^T)^T$, (1) can be converted into the following state equation:

$$\dot{x}= \left[\begin{array}{c|c} \multicolumn{2}{c}{a^T} \\ \hline h & F \end{array}\right] x + k_p c\{u+b^T w\} \qquad (5)$$

$$y = c^T x \qquad (6)$$

where a and b are unknown vectors depending on the coefficients of $\alpha(s)$ and $\beta(s)$, respectively. Note that the elements of a and b are given by

$$a_1 = f_2 - \alpha_1$$

$$a_i = (f_{i+1}-\alpha_i)-f_i a_1 \; ; \; f_{n+1}=0$$

$$b_i = \beta_i - f_i$$

$$i = 2,3,\cdots, n \tag{5.a}$$

Also, the n-dimensional vector c is defined as $c=(1,0,\cdots,0)^T$.

## Controller Design

In the model reference adaptive control design, the desired behaviour of the plant is expressed through the use of a reference model. Let the reference model be given by

$$\dot{x}_M = \begin{pmatrix} a_M^T \\ \hline h \;\vert\; F \end{pmatrix} x_M + k_M c r \triangleq A_M x_M + k_M c r \tag{7}$$

$$y_M = c_M^T x_M \tag{8}$$

where r(t) is the reference input, $k_M$ is a contant, and $a_M$ is predetermined such that $A_M$ has desired pole locations. Defining the state error and the output error as $e=x-x_M$ and $e_1=y-y_M$, eqns (5)-(8) lead to

$$\dot{e} = A_M e + k_p c\{u+\theta^T\psi\} \tag{9}$$

$$e_1 = c^T e \tag{10}$$

where

$$\theta = \left((a-a_M)^T/k_p,\; b^T,\; -k_M/k_p\right)^T \tag{9.a}$$

$$\psi = \left(x^T,\; w^T,\; r\right)^T \tag{9.b}$$

Now, to drive the state error to zero, the following adaptive control law is proposed based on the Lyapunov stability method:

$$u = -\hat{\theta}(t)^T \psi - z \tag{11}$$

$$\dot{\hat{\theta}} = \Gamma c^T P e \psi \tag{12}$$

$$z = \gamma c^T P e \triangleq \gamma \bar{z} \tag{13}$$

where $\hat{\theta}(t)$ is the estimate of $\theta$, and z is an error feedback signal introduced to increase the convergence rate of e. In (11) and (12), $\Gamma$ is a symmetric positive definite(spd) weighting matrix, and $\gamma > 0$ is a weighting factor. Also, P is the spd matrix satisfying

$$A_M^T P + P A_M = -Q \tag{14}$$

for a given spd matrix Q. Applying (11) to (9), the state error equation in closed loop becomes

$$\dot{e} = A_M e + k_p c \phi(t)^T \psi - k_p c z \tag{15}$$

where $\phi(t)=\theta-\hat{\theta}(t)$.

The overall error system is composed of eqns (15), (10), (12) and (13). The proof of stability directly follows by using a Lyapunov candidate

$$V = e^T P e + k_p \phi^T \Gamma^{-1} \phi \tag{16}$$

Computing the time derivative with respect to (12), (13) and (15), we get

$$\dot{V} = -e^T Q e - 2\gamma k_p \bar{z}^2 \leq 0 \tag{17}$$

Hence, e(t) and $\phi(t)$ are uniformly bounded, and furthermore e(t) converges to zero, as $t \to \infty$(1). In the Lyapunov synthesis, a positive value $\eta \triangleq -\dot{V}/V$ is regarded as a measure of the convergence rate(2). The larger $\eta$ is, the faster convergence speed becomes. In this sense, a sufficiently large valus of $\gamma$ forces $(-\dot{V})$ to increase resulting in fast convergence speed.

In stead of (13), the auxiliary input suggested by Lim and Eslami(3) may be used. In this case, (13) is replaced by

$$\dot{z} = -m(t)z + \gamma c^T P e \tag{18}$$

where $m(t)>0$ is an integral gain. Comparing to (18), (13) is computationally simple and avoids the complicated situation of finding an optimal m(t).

## A Numerical Example

Consider the second-order plant given by

$$(s^2+\alpha_1 s+\alpha_2)y(s)=k_p(s+\beta_2)u(s)$$

where all coefficients are unknown. Choosing $f(s)=s+4$, the example plant can be rewritten as

$$\dot{x} = \begin{pmatrix} \dot{y} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ 1 & -4 \end{pmatrix} x + k_p \begin{pmatrix} 1 \\ 0 \end{pmatrix} \{u+bw\}$$

$$\dot{w} = -4w + u$$

$$y = (1 \quad 0)\, x$$

Let us choose a reference model with poles at -3, -5 as

$$\dot{x}_M = \begin{pmatrix} -4 & 1 \\ 1 & -4 \end{pmatrix} x_M + \begin{pmatrix} 1 \\ 0 \end{pmatrix} r$$

$$y_M = (\,1 \quad 0\,)\, x_M$$

For simulation purposes, we use a set of values $\{\alpha_1=0,\ \alpha_2=-1,\ k_p=2,\ \beta_2=2\}$ or $\{a_1=4,\ a_2=-15,\ k_p=2,\ b=-2\}$. Using the proposed adaptive scheme, computer simulations are carried out for model reference adaptive control of the example plant. The results obtained under the following conditions are presented in Fig. 1 and 2:

r = step of height 5

$$\theta(0) = (1 \quad 1 \quad 1 \quad -1)^T$$

$$e(0) = (0 \quad 0)^T$$

$$\Gamma = \text{block diag}\ \{5\, I_2,\ 5,\ 2\}$$

$$\gamma = 10,\ Q = 30 I_2$$

where $I_2$ in the two-dimensional identity matrix. As can be seen in the figures, it is apparent that the existence of error feedback improves the transient characteristic with reduced input magnitude.

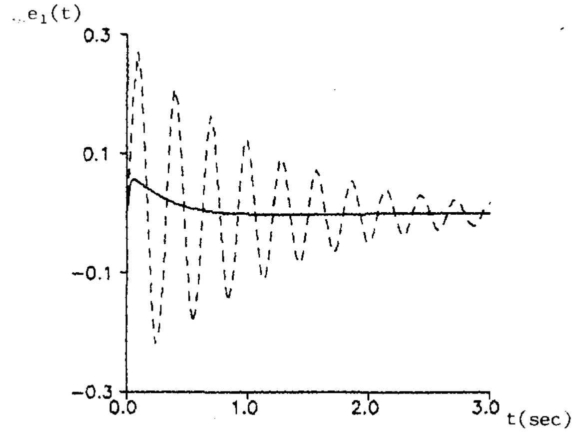


Fig. 1 Transient Behaviours of the output error ($e_1$): —— with error feedback, ---- without error feedback

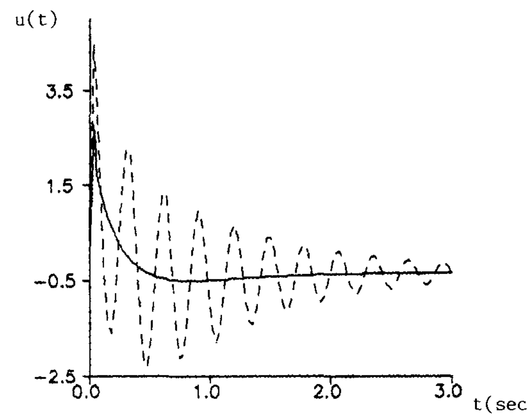


Fig. 2 Trajectories of the input (u): —— with error feedback, --- without error feedback

## Conclusion

It has been shown that by appending an error feedback term into the conventional adaptive controller, the transient performance of the resultant adaptive system can be improved. This better transient response and improved convergence speed provides a system with robust stability.

An extention study to the case when the relative order is greater than 2 is of immediate interest, and will be reported later.

## References


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3. K.Y. Lim and M. Eslami, "Adaptive Controller Designs for Robot Manipulator Systems Yielding Reduced Cartesian Error", IEEE Trans. Auto. Contr., Vol. 32, pp. 184-187, 1987.