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ON NONLINEAR ADAPTIVE CONTROL SYSTEMS INDEPENDENT OF THE DEGREE OF THE PROCESS

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Abstract: New design methods for constructing nonlinear adaptive control systems are considered. The proposed adaptive controllers are applicable to the case where the degree of the controlled process is unknown. It is shown that the degree of the controller is determined independently of the degree of the process. Several types of nonlinear functions are introduced to deal with uncertainties of the degree of the process. Finally, some simulation results show the effectiveness and simplicity of the proposed methods.

1. Introduction

In controlling processes whose physical and/or chemical characteristics are unknown, we usually derive linear stochastic models with suitable degrees and stochastic properties around operating points, and construct compensators for those process models. But the change of the driving conditions often induces the structural variations of the process model. In such cases, control design methods considering only one process model cannot be applicable.

Concerned with those problems, we have already proposed design methods of nonlinear adaptive control systems independent of the degree of the process (Miyasato *et al.* 1987 A, B), where two types of nonlinear functions (one for constructing adaptive sliding mide control systems and the other for constructing adaptive high-gain feedback control systems) are used to deal with uncertainty of degree, and the degree of the controller is determined independently of that of the controlled process. Those design methods have been shown to be applicable to the case where the degree of the process is unknown. In the present study, we show that other types of nonlinear functions also can be used and that various adaptive control systems are constructed according to the demand for the control system, such as perfect regulation of the output error and smoothness of the control input, etc. It is also shown that perfect regulation of the output error and continuous control inputs can be realized simultaneously in some cases. The effectiveness and simplicity of the proposed method are confirmed by some numerical experiments.

2. Problem Statement

We consider an unknown single-input, single-output nonlinear system modelled by the following equations:

$$\frac{d}{dt}x(t) = Ax(t)+bu(t)+f(x, u, t) \qquad (2\text{-}1)$$

$$y(t) = c^T x(t), \qquad (2\text{-}2)$$

where $A(\in R^{n\times n})$, b, $c(\in R^n)$ and degree n are assumed unknown. $f(x, u, t)$ is an unknown nonlinear term and/or disturbance and evaluated with known functions $h_i(\xi_i(t), t)$ as follows:

$$\|f(x, u, t)\| \le \sum_{i=1}^{k} M_i h_i(\xi_i(t), t), \qquad (0 \le M_i < \infty) \qquad (2\text{-}3)$$

where signals $\xi_i(t)$ are measurable, but constants M_i are unknown. Also the state $x(t)$ is assumed unknown.

The problem to be solved in this paper can be stated as follows: Given an unknown process with unknown degree, and a bounded known reference signal $y_M(t)$, determine a suitable controller such that the following equation holds.

$$\lim_{t\to\infty} e(t) = 0, \qquad (2\text{-}4)$$

where

$$e(t) = y_M(t) - y(t). \qquad (2\text{-}5)$$

3. Evaluation of State Variables

Before constructing adaptive control systems, several assumptions are introduced.

< Assumption 1 >

(c, A) is observable.

< Assumption 2 >
$c^T b \neq 0$

< Assumption 3 >
$h_i(\xi_i(t), t)$ have following properties.
$\exists N_0, N_1 \quad (0 \leq N_0 < \infty,\ 0 < N_1 < \infty)$
$h_i(\xi_i(t), t) \leq N_0 + N_1 \sup_{t \geq \tau} |y(\tau)| \qquad (1 \leq i \leq k)$

< Assumption 4 >
$\{ s : c^T (sI-A)^{-1} b = 0 \} \in C^-$

< Assumption 5 >
A reference signal $y_M(t)$ is uniformly bounded and C^1.

Assumption 1, 2, and 4 assert that the linear part of the process is observable, with relative degree one, and has stable inverse. Under Assumption 1 and 2, the input-output representation of the process is obtained as follows:

$$\begin{aligned}\frac{d}{dt}y(t)+\lambda_0 y(t) &= \theta_1^T(\lambda_0 I+F)v_{1f}(t)+\theta_1^T g u_f(t)+\theta_2^T v_2(t)\\ &+\theta_3^T v_3(t)+\theta_4 y(t)+\theta_5^T f(x, u, t)\\ &+\theta_0 u(t), \end{aligned} \tag{3-1}$$

where

$$\frac{d}{dt}u_f(t) = -\lambda_0 u_f(t)+u(t) \qquad (\lambda_0 > 0) \tag{3-2}$$

$$\begin{aligned}\frac{d}{dt}v_{1f}(t) &= F v_{1f}(t)+g u_f(t)\\ \frac{d}{dt}v_2(t) &= F v_2(t)+g y(t)\\ \frac{d}{dt}v_3(t) &= F v_3(t)+T f(x, u, t), \end{aligned} \tag{3-3}$$

$$\theta_0 = c^T b \tag{3-4}$$

and (F, g) is a n-dimensional controllable pair with a stable matrix F; T is a transformation matrix; parameters θ_i (with suitable dimensions) are determined properly (for details, see Miyasato *et al.* 1987 A, B and Morse 1980). Concerned with equation (3-1), following state variable filters with degree one are introduced.

$$\begin{aligned}\frac{d}{dt}v_{1f0}(t) &= -\lambda_f v_{1f0}(t)+|u_f(t)|\\ \frac{d}{dt}v_{20}(t) &= -\lambda_f v_{20}(t)+|y(t)|\\ \frac{d}{dt}v_{3i0}(t) &= -\lambda_f v_{3i0}(t)+h_i(\xi_i(t), t), \qquad (1 \leq i \leq k)\end{aligned} \tag{3-5}$$

where λ_f is determined such that

$$\|\exp Ft\| \leq M_0 \exp(-\lambda_f t). \qquad (0 < M_0 < \infty,\ \lambda_f > 0) \tag{3-6}$$

Then $v_{1f}(t)$, $v_2(t)$, and $v_3(t)$ in equation (3-1) are evaluated as follows:

$$\begin{aligned}\|v_{1f}(t)\| &\leq M_0 \|g\| v_{1f0}(t)\\ \|v_2(t)\| &\leq M_0 \|g\| v_{20}(t)\\ \|v_3(t)\| &\leq M_0 \sum_{i=1}^{k} \|T\| M_i v_{3i0}(t)\end{aligned} \tag{3-7}$$

4. Nonlinear Adaptive Control System

The proposed nonlinear adaptive control systems are stated as follows:

<u>*Control Laws*</u>:

$$\begin{aligned}u(t) = \{&\hat{\varphi}_0(t)\,|\tfrac{d}{dt}y_M(t)+\lambda_0 y_M(t)|+\hat{\varphi}_1(t)v_{1f0}(t)\\ &+\hat{\varphi}_2(t)v_{20}(t)+\sum_{i=1}^{k}\hat{\varphi}_{3i}v_{3i0}(t)+\hat{\varphi}_4(t)|y(t)|\\ &+\sum_{i=1}^{k}\hat{\varphi}_{5i}h_i(\xi_i(t), t)+\hat{\varphi}_6|u_f(t)|\}(sgn\,\theta_0)\,\alpha(e(t))\end{aligned} \tag{4-1}$$

<u>*Adaptive Laws*</u>:

$$\begin{aligned}\frac{d}{dt}\hat{\varphi}_0(t) &= g_0\,|\tfrac{d}{dt}y_M(t)+\lambda_0 y_M(t)|\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_1(t) &= g_1\,v_{1f0}(t)\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_2(t) &= g_2\,v_{20}(t)\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_{3i}(t) &= g_{3i}\,v_{3i0}(t)\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_4(t) &= g_4\,|y(t)|\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_{5i}(t) &= g_{5i}\,h_i(\xi_i(t), t)\,\beta(e(t))\,\gamma(e(t))\\ \frac{d}{dt}\hat{\varphi}_6(t) &= g_6\,|u_f(t)|\,\beta(e(t))\,\gamma(e(t)).\\ &(1 \leq i \leq k;\ g_j, g_{im} > 0)\end{aligned} \tag{4-2}$$

where $\alpha(e)$, $\beta(e)$ and $\gamma(e)$ are nonlinear functions to be determined in the followings:

Case (1)
$\alpha(e) = sgn(e)$
$\beta(e) = 1$
$\gamma(e) = |e|$

Case (2)
$\alpha(e) = \frac{e}{|e|+\delta}$
$\beta(e) = \begin{cases} 1 & (|e| \geq \varepsilon^*) \\ 0 & (|e| < \varepsilon^*). \end{cases}$
$\gamma(e) = \alpha(e)\,e$, or $|e| \qquad (\delta, \varepsilon^* > 0)$

Case (3)

$$\alpha(e) = \begin{cases} sgn(e) & (|e| \geq \delta) \\ \frac{e}{\delta} & (|e| < \delta), \end{cases}$$

$$\beta(e) = \begin{cases} 1 & (|e| \geq \varepsilon^*) \\ 0 & (|e| < \varepsilon^*), \end{cases}$$

$$\gamma(e) = |e| \qquad (\delta, \varepsilon^* > 0)$$

Case (4)

$$\alpha(e) = \begin{cases} sgn(e) & (|e| \geq \varepsilon) \\ \frac{e}{\varepsilon} & (|e| < \varepsilon), \end{cases}$$

$$\beta(e) = 1$$

$$\gamma(e) = \alpha(e)\, e$$

$$\frac{d}{dt}\varepsilon = -l\varepsilon, \qquad \varepsilon(0) = \varepsilon^* > 0, \qquad l > 0$$

A design parameter ε^* in *Case* (2), (3) is set as the admissible magnitude of $|e|$. Other parameters δ, g_i, g_{im} (>0) are chosen arbitrarily. Now the main theorem of this paper is stated as follows:

[Theorem]

Consider a controlled process with control laws and adaptive laws described by (2-1), (2-2), (3-5), (4-1), and (4-2). Suppose Assumption 1 ~ 5 can be met, then the resulting control system is uniformly bounded. Furthermore, the tracking error $e(t)$ converges to zero asymptotically, when nonlinear functions in Case (1) or Case (4) are used. In consrast to those, the tracking error $e(t)$ converges to a region defined by

$$S(\varepsilon^*) = \{e \; ; \; |e| < \varepsilon^*\} \tag{4-3}$$

in Case (2) and (3).

< Proof >

We set the functions $V_i(t)$ (i=1, 2) as follows:

$$V_1(t) = e(t)^2 + |\theta_0| [\{\varphi_0 - \hat{\varphi}_0(t)\}^2/g_0 + \{\varphi_1 - \hat{\varphi}_1(t)\}^2/g_1 + \{\varphi_2 - \hat{\varphi}_2(t)\}^2/g_2 + \sum_{i=1}^{k}\{\varphi_{3i} - \hat{\varphi}_{3i}(t)\}^2/g_{3i} + \{\varphi_4 - \hat{\varphi}_4(t)\}^2/g_4 + \sum_{i=1}^{k}\{\varphi_{5i} - \hat{\varphi}_{5i}(t)\}^2/g_{5i} + \{\varphi_6 - \hat{\varphi}_6(t)\}^2/g_6] \tag{4-4}$$

$$V_2(t) = \frac{\beta_0}{l}\varepsilon, \qquad (\beta_0 > 0) \tag{4-5}$$

where φ_i, φ_{ij} are some constant numbers to be determined later.

Case (1)

Considering control laws and adaptive laws. we take the time derivative of $V_1(t)$ and set the parameters φ_i, φ_{ij} as follows:

$$\varphi_0 \geq 1/|\theta_0|, \qquad \varphi_1 \geq \|\theta_1^T(\lambda_0 I + F)\| M_0 \| g \| / |\theta_0|$$
$$\varphi_2 \geq \|\theta_2\| M_0 \| g \| / |\theta_0|, \qquad \varphi_{3i} \geq \|\theta_3\| M_0 \| g \| M_i / |\theta_0|$$
$$\varphi_4 \geq |\theta_4| / |\theta_0|, \qquad \varphi_{5i} \geq \|\theta_5\| M_i / |\theta_0|$$
$$\varphi_6 \geq |\theta_1^T g| / |\theta_0|. \tag{4-6}$$

Then

$$\frac{1}{2}\frac{d}{dt}V_1(t) \leq -\lambda_0 e(t)^2 \leq 0. \tag{4-7}$$

Case (2)

a) $\gamma(e) = \alpha(e)e$

In case that $|e(t)| \geq \varepsilon^*$, we take the time derivative of $V_1(t)$, and set φ_i, φ_{ij} as follows:

$$\varphi_0 \geq (1 + \frac{\delta}{\varepsilon^*}) / |\theta_0|$$
$$\varphi_1 \geq (1 + \frac{\delta}{\varepsilon^*}) \|\theta_1^T(\lambda_0 I + F)\| M_0 \| g \| / |\theta_0|$$
$$\varphi_2 \geq (1 + \frac{\delta}{\varepsilon^*}) \|\theta_2\| M_0 \| g \| / |\theta_0|$$
$$\varphi_{3i} \geq (1 + \frac{\delta}{\varepsilon^*}) \|\theta_3\| M_0 \| g \| M_i / |\theta_0|$$
$$\varphi_4 \geq (1 + \frac{\delta}{\varepsilon^*}) |\theta_4| / |\theta_0|, \qquad \varphi_{5i} \geq (1 + \frac{\delta}{\varepsilon^*}) \geq \|\theta_5\| M_i / |\theta_0|$$
$$\varphi_6 \geq (1 + \frac{\delta}{\varepsilon^*}) |\theta_1^T g| / |\theta_0|. \tag{4-8}$$

Then

$$\frac{1}{2}\frac{d}{dt}V_1(t) \leq -\lambda_0 e(t)^2 \leq 0, \tag{4-9}$$

in case that $|e(t)| \leq \varepsilon^*$.

b) $\gamma(e) = |e|$

Except that g_i, g_{ij} are divided as follows:

$$g_i = g_i^1 g_i^2$$
$$g_{ij} = g_{ij}^1 g_{ij}^2 \tag{4-10}$$

and that the following function $V_1(t)^*$ is utilized.

$$V_1(t)^* = e(t)^2 + |\theta_0| [\{\varphi_0 - \hat{\varphi}_0(t)\}^2/g_0^1 + \{\varphi_1 - \hat{\varphi}_1(t)\}^2/g_1^1 + \{\varphi_2 - \hat{\varphi}_2(t)\}^2/g_2^1 + \sum_{i=1}^{k}\{\varphi_{3i} - \varphi_{3i}(t)\}^2/g_{3i}^1 + \{\varphi_4 - \hat{\varphi}_4(t)\}^2/g_4^1 + \sum_{i=1}^{k}\{\varphi_{5i} - \hat{\varphi}_{5i}(t)\}^2/g_{5i}^1 + \{\varphi_6 - \hat{\varphi}_6(t)\}^2/g_6^1] \tag{4-11}$$

the proof is carried out almost same manner as

Case (2) a). By setting

$$\varphi_0 g_0^2 \geq 1/|\theta_0|, \qquad \varphi_1 g_1^2 \geq \|\theta_1^T(\lambda_0 I+F)\|M_0\|g\|/|\theta_0|$$
$$\varphi_2 g_2^2 \geq \|\theta_2\|M_0\|g\|/|\theta_0|, \qquad \varphi_{3i} g_{3i}^2 \geq \|\theta_3\|M_0\|g\|M_i/|\theta_0|$$
$$\varphi_4 g_4^2 \geq |\theta_4|/|\theta_0|, \qquad \varphi_{5i} g_{5i}^2 \geq \|\theta_5\|M_i/|\theta_0|$$
$$\varphi_6 g_6^2 \geq |\theta_1^T g|/|\theta_0|. \tag{4-12}$$
$$0 < g_i^2 \leq 1/(1+\frac{\delta}{\varepsilon^*})$$
$$0 < g_{ij}^2 \leq 1/(1+\frac{\delta}{\varepsilon^*}),$$

it follows

$$\frac{1}{2}\frac{d}{dt}V_1(t)^* \leq -\lambda_0 e(t)^2 \leq 0. \tag{4-13}$$

in case that $|e(t)| \leq \varepsilon^*$.

Case (3)

If $\delta<\varepsilon^*$, the proof is carried out almost same way as *Case* (1). If $\delta\geq\varepsilon^*$, the proof is carried out almost same way as *Case* (2) b), except that φ_i, φ_{ij} are set as (4-12), and that g_i^2, g_{ij}^2 are set as follows:

$$0 < g_i^2 \leq \frac{\varepsilon^*}{\delta}$$
$$0 < g_{ij}^2 \leq \frac{\varepsilon^*}{\delta} \tag{4-14}$$

Then in case that $|e(t)| \geq \varepsilon^*$,

$$\frac{1}{2}\frac{d}{dt}V_1(t)^* \leq -\lambda_0 e(t)^2 \leq 0. \tag{4-15}$$

Case (4)

In case that $|e(t)| \geq \varepsilon$, the proof is almost same as *Case* (1) except that the function $\{V_1(t)+V_2(t)\}$ is utilized for stability analysis. In case that $|e(t)| < \varepsilon$, for β_0 such that

$$\beta_0 \geq N_0+N_1\sup_t|y(t)|+N_2\sup_t\left|\frac{d}{dt}y_M(t)+\lambda_0 y_M(t)\right|$$
$$0 \leq N_0 < \infty, \quad 0 < N_1, \quad N_2 < \infty, \tag{4-16}$$

the following result is derived.

$$\frac{1}{2}\frac{d}{dt}\{V_1(t)+V_2(t)\} \leq -\lambda_0 e(t)^2 \leq 0 \tag{4-17}$$

(End of proof)

5. Simulation Results

Numerical simulation studies are performed to show the effectiveness of the proposed methods. As one example, the method in *Case* (2) a) is applied to a process described as follows:

$$A = diag(-\lambda_1, -\lambda_2, \cdots, -\lambda_n)$$
$$b = [1\ 1 \cdots 1\ 0]^T$$
$$c = [c_1, c_2, \cdots, c_n]^T$$
$$f(x, u, t) = [0\ 0 \cdots 0\ 1]^T 0.5 \sin y$$
$$\lambda_i = 0.1\,(i-5), \qquad c_i = 1/i \qquad (1 \leq i \leq n)$$
$$n = 15,\ 50$$

In this case, $h_1 = 1$ $(k=1)$, so that no exact knowledge about the nonlinear term of the process is needed. A reference signal which the output of the process must track is

$$y_M(t) = \sin(\pi t/2).$$

Design parameters are set as follows:

$$g_i = g_{ij} = 3, \qquad \lambda_0 = 1, \qquad \lambda_f = 2$$
$$\delta = 0.001, \qquad \varepsilon^* = 0.001.$$

Fig. 1 and Fig. 2 show the results where the degree of the process is set to be $n = 15$, and $n = 50$ respectively. In each case, the proposed adaptive controller is identical and with 4 degrees and 7 control parameters; those constants (4 and 7) are determined independently of the degree of the process (n).

6. Concluding Remarks

In the present study, we propose several methods for constructing nonlinear adaptive control systems whose degrees are independent of that of the controlled process. In *Case* (1), a kind of adaptive sliding mode control system is constructed, where discontinuous control input is utilized. In other cases, continuous control inputs are used to construct a kind of high gain feedback control system. Perfect regulation of the output error can be realized in *Case* (1) and (4). Contrary to those, in *Case* (2) and (3), perfect regulation cannot be realized, but the magnitude of the output error can be made small freely. Especially in *Case* (4), perfect regulation can be realized using continuous control inputs. In each case, the degree of the controller is equal to $5+2k$, and it is determined not from the degree of the controlled process, but from how the nonlinear term and/or disturbance are evaluated in (2-3). One drawback of these methods is thought that their effectiveness is restricted to the case where the relative degree of the process is equal to one only. Extension of those methods to the general case where the relative degree is greater than one, is left for our future study.

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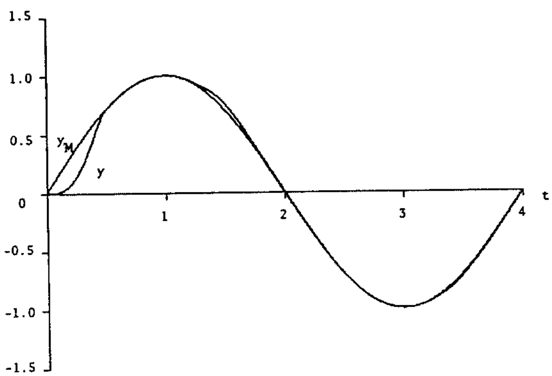
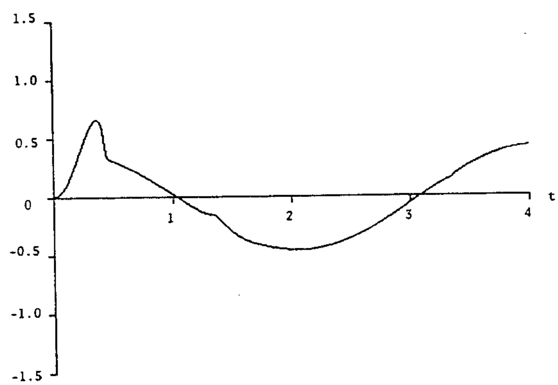
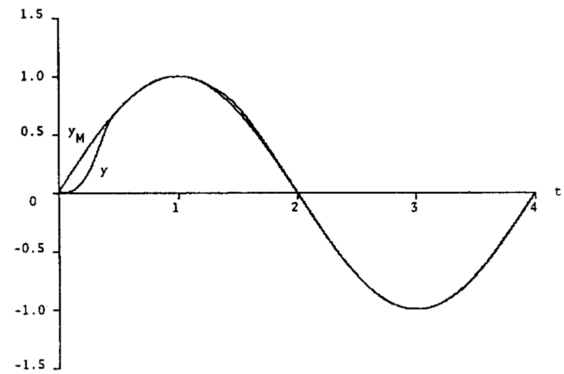


a) Response curves

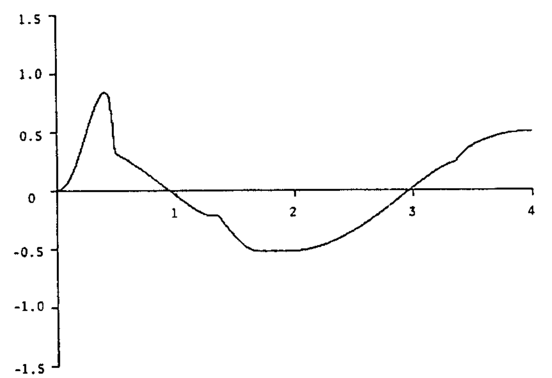


b) Control input

Figure 1 Simulation results ($n = 15$)



a) Response curves



b) Control input

Figure 2 Simulation results ($n = 50$)