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OPTIMIZATION PROBLEMS IN ELECTRIC POWER SYSTEM

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1. Introduction

This paper provides a brief summary of the current state of the art of mathematical programming techniques for optimization problems in power systems. The optimization problems surveyed here are the optimal power flow (OPF), unit commitment and optimal reactive power planning (ORPP) problem. Various optimization techniques described are the recursive quadratic programming (RQP) exploiting an exact or approximate Hessian matrix, the variable reduction technique, the mixed integer programming, the subgradient method, the Lagrange relaxation technique, the variable metric method, and the Benders decomposition.

The special features of the optimization problem of a power system are 1) Large Scale, 2) Real Time, 3) Nonlinear, 4) Nonconvex, 5) Mixed Integer. Many algorithms have been proposed for solving optimization problems by exploiting these special properties. For example, a large scale optimal power flow problem with 4,000 nonlinear constraints and 6,000 variables has been solved.

This paper concludes that mathematical programming is an important tool in the field of electric power system.

2.1 Optimal Power Flow

The Optimal Power Flow (OPF) problem is formulated as follows:

$$\text{Minimize } F(x) = \sum_{i \in G} f_i(P_i) \qquad (1)$$

$$\text{sub. to } \underline{P}_i \le P_i \le \overline{P}_i, \; i \in G \qquad (2)$$

$$\underline{Q}_i \le Q_i \le \overline{Q}_i, \; i \in N \qquad (3)$$

$$\underline{V}_i \le V_i \le \overline{V}_i, \; i \in N \qquad (4)$$

$$\underline{n}_k \le n_k \le \overline{n}_k, \; k \in K \qquad (5)$$

$$P_i = p_i, \; i \in L \qquad (6)$$

where G, L and K respectively denotes an index set of generator nodes, load nodes and load ratio transformers, and $N = G \cup L$. The function f_i in (1) represents a fuel cost as a function of generator output. Equation (2) is a limit on the generator output. Equations (3)-(5) are the upper and lower limits on reactive power, voltage magnitude and transformer tap ratio, respectively. Equation (6) denotes a active power balance at load nodes.

In these equations, independent variables are V_i, θ_i and n_k, the functions P_i and Q_i are nonlinear function of these independent variables.

2.2 Recent Study on OPF

1) Burchett, et al.[15]

The OPF problem with 1,500 variables is solved on DEC-VAX 11/780 within 60 minutes. The optimization software used is MINOS/Augmented developed at Stanford University.

2) Talukdar, et al.[17]

The dimension of the Hessian matrix is reduced by using the fact that the equation (6) is an equality constraint.

3) Happ, et al.[19]

The RQP method with an exact Hessian matrix is employed, and a large scale nonconvex QP problem is solved repeatedly.

4) Sun, et al.[20]

Since the Kuhn-Tucker conditions for a nonlinear programming problem with equality constraints are expressed as a system of nonlinear equations, the Newton method for finding a solution of nonlinear equations is employed.

5) Aoki, et al.[6]

The equations (5),(6) are approximated by a quadratic function, and the Hessian of the Lagrange function for the OPF problem is approximated by a constant matrix. So the QP problem in the RQP method can be solved efficiently by exploiting the sparsity.

3.1 Unit Commitment

The Unit Commitment problem can be formulated as follows:

$$\text{Min.} \quad \sum_{t=1}^{24} \sum_{i=1}^{I} \{f_i(P_{it}) + S_i(x_{it}, u_{it})\} \qquad (7)$$

$$\text{sub. to} \quad \sum_i P_{it} = L_t \qquad (8)$$

$$\sum_i \overline{P}_i \cdot u_{it} \geq R_t \qquad (9)$$

$$\underline{P}_i \cdot u_{it} \leq P_{it} \leq \overline{P}_i \cdot u_{it} \qquad (10)$$

$$u_{it} = 1 \text{, if } 0 < x_{it} < mut(i) \qquad (11)$$

$$u_{it} = 0 \text{, if } mdt(i) < x_{it} < 0 \qquad (12)$$

where I : number of generators,

$f_i(P_{it})$: fuel cost function,

S_i : start up/shutdown cost,

P_{it} : output of generator-i at period-t,

L_t, R_t : demand and reserve,

$\overline{P}_i$, $\underline{P}_i$: upper and lower limit,

x_{it} : state variable,

u_{it} : control variable,

$u_{it}=1(0)$ if up(down) decision.

$mut(mdt)$: minimum up(down) time.

3.2 Recent Study on Unit Commitment

1) Merlin, et al.[22]

The Lagrange relaxation method is applied and the subgradient method is used for maximizing the dual function. The E.D.F. system with 172 generators model for 48 hours is solved.

2) Bertsekas, et al.[23]

The unique approximation method using an exponential function is proposed, and a system with 100 generators is solved.

3) Aoki, et al.[11]

The Shor's ellipsoid method is employed for maximizing the dual function, and a large scale unit commitment problem including the fuel-constrained thermal units and the pumped-storage hydro units are solved.

4.1 Optimal Reactive Power Planning

The Optimal Reactive Power Planning (ORPP) problem is formulated as follows:

$$\text{Min.} \quad \sum_{i \in L_N} d_i c_i y_i \qquad (13)$$

$$\text{sub. to} \quad \underline{P}_{it} \leq P_{it} \leq \overline{P}_{it} \text{, } i \in G, \ t \in T \qquad (14)$$

$$\underline{Q}_{it} \leq Q_{it} \leq \overline{Q}_{it} \text{, } i \in N, \ t \in T \qquad (15)$$

$$\underline{V}_{it} \leq V_{it} \leq \overline{V}_{it}\ ,\ i\epsilon N,\ t\epsilon T \quad (16)$$

$$P_{it} = p_{it}\ ,\ i\epsilon L,\ t\epsilon T \quad (17)$$

$$\underline{n}_{kt} \leq n_{kt} \leq \overline{n}_{kt}\ ,\ k\epsilon K,\ t\epsilon T \quad (18)$$

$$Q_{it} = q_{it} + V_{it}^2 c_{it} y_{it}\ ,\ i\epsilon L,\ t\epsilon T \quad (19)$$

$$Y_t = Y_t(n_{1t},\ldots,n_{kt}) \quad (20)$$

where $L = L_N \cup L_E$, L_N and L_E are the index set of candidate nodes for capacitor installation and capacitor existing nodes, respectively.

4.2 Recent Study on ORPP

1) Happ, et al.[28]

In their approach, an integer variable is considered as a continuous one, and an obtained value is rounded to a nearest integer value. The continuous problem is solved by using MINOS/Augmented.

2) Iyer, et al.[31]

They employ the recursive mixed-integer programming method, and the Benders decomposition technique is applied to each mixed-integer programming problem. The decomposed subproblems are solved by using the branch-and-bound method.

3) Lebow, et al.[29]

They use the generalized Benders decomposition, and a 2-stage hierarchical computation method is developed by using the restriction technique.

4) Granville, et al.[30]

The 2-stage structure is expanded to the 3-stage one, and the IEEE 118 node system with 3 load levels and 4 contingency cases is solved.

5) Aoki, et al.[33]

A problem with a large number of load levels and contingencies is solved by using a decomposition method for a convex quadratic programming problem. In this approach, the recursive quadratic programming method is applied for solving the continuous reactive power planning problem, and the continuous problem is decomposed by fixing the coordinate variables.

5. Conclusion

Three types of the optimization problems in power systems are surveyed, and various optimization technique applied for solving them are described briefly. It may be concluded that the mathematical programming technique is an efficient tool for optimization problem in power systems.

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