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마르코프 리뉴얼 프로세스 모델을 이용한 음성 및 데이타 다중화기의 성능분석

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Performance Analysis of a Voice/Data Multiplexer Based on Markov Renewal Process Modeling

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Abstract

Voice and data transmission on a common link is modeled as a Markov renewal process. The performance of an integrated voice/data multiplexer with an arbitrary message length distribution and the non-gated service discipline, particularly the mean data message delay, is analyzed using the model.

1. Introduction

Integrated voice/data transmission system has usually been modeled as either a two-dimensional Markov process in the continuous-time domain [1] or a two-dimensional Markov chain in the discrete-time domain [2], [3]. Various performance measures of the system have been obtained through these models. There are, however, some restrictions in these approaches. When the two-dimensional Markov process model is used, the data message length is always assumed to be exponentially distributed. A system with any other message length distribution cannot be analyzed using the model. However, this restriction can be resolved by using a two-dimensional Markov chain model in the discrete-time domain, where data messages with arbitrary length distribution are segmented into fixed length packets, and each of these packets is transmitted in a slot. Nevertheless, this model may be applied to a system with the gated service discipline only, in which data messages arrived in a frame are momentarily blocked and are served in the succeeding frames. The performance of a system with the non-gated service discipline may be estimated using the same discrete-time domain analysis. However, the performance difference between the gated service and the non-gated service is rather significant, as will be shown later.

In this paper, an integrated voice/data multiplexer is modeled as a Markov renewal process, and the data buffer behavior of the system with an arbitrary message length distribution and the non-gated service discipline is analyzed in the continuous-time domain.

2. Model Description and Analysis

We now analyze the performance of a voice/data multiplexer and obtain moments of the data queue length at the data message departure instants. We assume that the link consists of M channels and its capacity is C bits/s. Also, we assume that K channels among M channels ($K < M$) are dedicated to data only, and the remaining S ($S = M-K$) channels are shared by voice and data with voice having priority over data. Speech activity is monitored, and only talkspurts are transmitted. Talkspurts arriving at the system with S channels already occupied by other talkspurts are lost. Data messages are queued and transmitted through the dedicated K channels and the shared channels unused by voice. If we assume that silence and talkspurt durations are exponentially distributed, the number of talkspurts in the system can be regarded as a birth and death process with the generator $\mathbf{T}$ in the form of an $(N+1)$-by-$(N+1)$ tridiagonal matrix, where N is the smaller number between L and S [5].

$\mathbf{T}$ is simply expressed in terms of the mean talkspurt and silence durations. Data messages arrive according to a Poisson process with the arrival rate λ, and have an arbitrary message length distribution $F(x)$. If there are i talkspurts at the time when the transmission of a data message starts, $M-i$ channels out of M channels would be used to transmit the data message. Then, the conditional data message service time distribution $H_i(t)$ would be $F(C(M-i)t/M)$.

Let D_n and J_n be the number of data messages in queue and the number of talkspurts at the nth data message departure point, respectively, and let X_n be the time interval between the $(n-1)$th and nth data message departure points. Then, the process $\{D_n, J_n, X_n; n=1, 2, \cdots\}$ forms a time-homogeneous Markov renewal process with the kernel [5]

$$\mathbf{Q}(x) = \begin{bmatrix} \mathbf{B}_0(x) & \mathbf{B}_1(x) & \mathbf{B}_2(x) & \cdots \\ \mathbf{A}_0(x) & \mathbf{A}_1(x) & \mathbf{A}_2(x) & \cdots \\ 0 & \mathbf{A}_0(x) & \mathbf{A}_1(x) & \cdots \\ 0 & 0 & \mathbf{A}_0(x) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \quad (1)$$

where $\mathbf{A}_\nu(x)$ and $\mathbf{B}_\nu(x)$, $\nu = 0, 1, 2, \cdots$, are $(N+1)$-by-$(N+1)$ matrices, and the element of the $[k(N+1)+i]$th row and $[l(N+1)+j]$th column of $\mathbf{Q}(x)$ represents the probability $Pr(D_{n+1} = l,\ J_{n+1} = j,\ X_{n+1} \leq x \mid D_n = k, J_n = i)$
Here, $\mathbf{A}_\nu(x)$ and $\mathbf{B}_\nu(x)$ are given by

$$\mathbf{A}_\nu(x) = \int_0^x \frac{e^{-\lambda y}(\lambda y)^\nu}{\nu !} d\mathbf{A}(y), \quad (2)$$

$$\mathbf{B}_\nu(x) = \int_0^x \mathbf{E}(x-t)\mathbf{A}_\nu(t)dt, \quad (3)$$

where

$$\mathbf{A}(y) = \sum_{i=1}^{N+1} \int_0^y \mathbf{I}_i e^{t\mathbf{T}} dH_i(t), \quad (4)$$

$$\mathbf{E}(x) = \int_0^x e^{t\mathbf{T}} \lambda e^{-\lambda t} dt. \quad (5)$$

$\mathbf{I}_i$ is an $(N+1)$-by-$(N+1)$ matrix in which the ith diagonal element is 1 and all others are 0.

The above quantities may be interpreted as follows.

$[A_\nu(x)]_{ij} = Pr(J_{n+1}=j,\ X_{n+1} \leq x$, *data message arrivals in* $X_{n+1}=\nu \mid J_n=i,\ D_n \neq 0)$

$[B_\nu(x)]_{ij} = Pr(J_{n+1}=j,\ X_{n+1} \leq x$, *data message arrivals in* $X_{n+1}=\nu+1 \mid J_n=i,\ D_n=0)$

$A_{ij}(x) = Pr(J_{n+1}=j,\ X_{n+1} \leq x \mid J_n=i)$

$E_{ij}(x) = Pr($*data idle period ends with* j *talkspurts, data idle period* $\leq x$ | *data idle period starts with* i *talkspurts*$)$.

Let $\mathbf{Q} \triangleq \lim_{x \to \infty} \mathbf{Q}(x)$ and $\mathbf{r}$ be the steady state probability distribution of $\mathbf{Q}$ such that $\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \cdots)$, where $\mathbf{r}_i$, $i = 0, 1, 2, \cdots$, is a row vector with $N+1$ elements, $r_{i0}, r_{i1}, \cdots, r_{iN}$. Then, r_{ij}, $j = 0, 1, 2, \cdots, N$, is the steady-state probability that there are i data messages queued and j active talkspurts.

The stationary equation $\mathbf{rQ} = \mathbf{r}$ may be written, from (1), as

$$\mathbf{r}_i = \mathbf{r}_0\mathbf{B}_i + \sum_{k=1}^{i+1} \mathbf{r}_k \mathbf{A}_{i-k+1}, \quad i=1, 2, \cdots \quad (6)$$

where $\mathbf{A}_i = \lim_{x \to \infty} \mathbf{A}_i(x)$ and $\mathbf{B}_i = \lim_{x \to \infty} \mathbf{B}_i(x)$.

Defining $\mathbf{R}(z) \triangleq \sum_{i=0}^{\infty} \mathbf{r}_i z^i$, we obtain from (2), (3), (4), (5) and (6),

$$\mathbf{R}(z)[z\mathbf{I} - \tilde{\mathbf{A}}(\lambda-\lambda z)] = \mathbf{r}_0(\mathbf{E}z - \mathbf{I})\tilde{\mathbf{A}}(\lambda-\lambda z) \quad (7)$$

where $\tilde{\mathbf{A}}(\cdot)$ is the Laplace transform of $\mathbf{A}(t)$ and $\mathbf{E} = \lim_{x \to \infty} \mathbf{E}(x)$, and they are given, respectively, by

$$\tilde{\mathbf{A}}(\lambda-\lambda z) = \sum_{i=1}^{N+1} \mathbf{I}_i \sum_{k=1}^{N+1} \mathbf{B}_k \tilde{F}\left(\frac{M(\lambda-\lambda z-\delta_k)}{(M-i)C}\right), \quad (8)$$

$$\mathbf{E} = \sum_{k=1}^{N+1} \mathbf{B}_k \frac{\lambda}{\lambda-\delta_k}. \quad (9)$$

In the above equations, δ_k's are eigenvalues of $\mathbf{T}$, $\mathbf{B}_k$'s are the spectral components of $\mathbf{T}$, i.e., $\mathbf{T} = \sum_{k=1}^{N+1} \delta_k \mathbf{B}_k$, and $\tilde{F}(\cdot)$ is the Laplace transform of $F(x)$.

Various moments of the data queue length can be obtained by differentiating (7) and letting $z=1$. $\mathbf{r}_0$ can be claculated in the same way as was done by Neuts [4]. Once the mean queue length has been calculated, the mean message delay can be obtained using the Little's formula [4].

3. Numerical Results and Discussion

The proposed method has carefully been validated through simulation and comparison with the results of previous studies. In our analysis and simulation, mean durations of talkspurt and silence were 1.2 s and 1.8 s, respectively. It was assumed that each channel can transmit on the average 100 data messages per second. This corresponds to the case that the mean data message length is 560 bits, providing that each channel capacity is 56 kbits/s. For the exponential message length distribution case, we calculated the mean message delay using our Markov renewal process modeling technique and the two-dimensional Markov process modeling approach [1]. In both methods, the non-gated service discipline was used. The two results came out to be very close as expected. In Fig. 1 the mean data message delay is shown for various values of L, S and M. The curves show the mean message delay versus data capacity utilization ρ_d defined by $\rho_d \triangleq M\lambda\bar{l}/C(M-\bar{v})$, where $\bar{l}$ is the mean data message length and $\bar{v}$ is the mean number of talkspurts. We can see the effect of multiplexing in this figure. For the same data capacity utilization, the mean data message delay decreases as the system size increases.

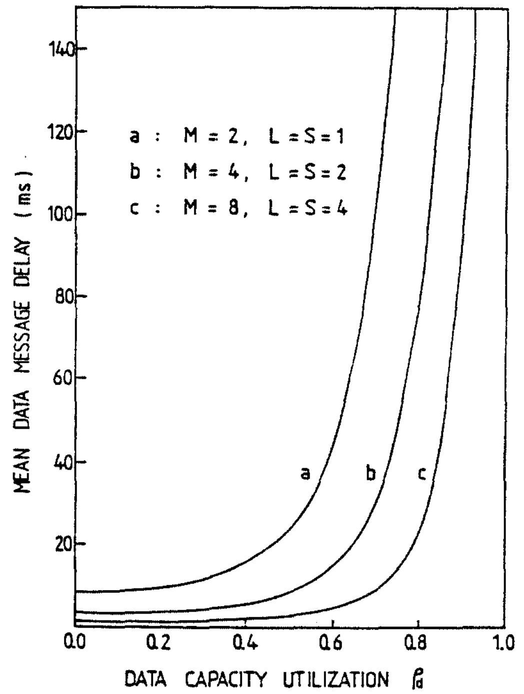

그림 1 데이타 메시지의 길이 분포가 지수 함수일 때, 데이타 용량 이용도 대비 평균 데이타 메시지 지연 시간.

Fig. 1 Mean data message delay vs. data capacity utilization, when the message length distribution is exponential.

In Fig. 2 we show the mean data delay when the data messages have the same length. This result is validated by simulation, and is compared with the result of Sriram et al. that was obtained using the two-dimensional Markov chain model in the discrete-time domain and assuming the gated service discipline [2]. We can see a rather large difference between our result and theirs, possibly due to different service disciplines. Another reason of discrepancy is that Sriram's result is based on the statistics at the frame boundary where the queue size is largest. Accordingly, the mean message delay obtained from this mean queue length is slightly overestimated. Note that the two curves come closer as ρ_d increases.

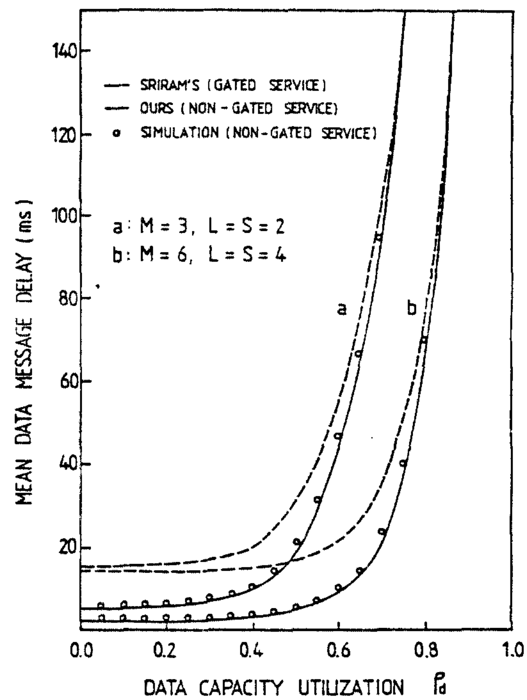

그림 2 데이타 메시지의 길이가 일정할 때, 데이타 용량 이용도 대비 평균 데이타 메시지 지연 시간.

Fig. 2 Mean data message delay vs. data capacity utilization, when the message length is fixed.

To see the applicability of our continuous-time domain analysis to time division multiplexing systems, we have simulated a time division multiplexer with the non-gated service discipline. In this simulation, the frame size was set to 10 ms as in [2]. The result is shown also in Fig. 2, which agrees closely with our analysis.

Finally, it is worthwhile to mention that this new modeling technique can be used to analyze other mechanisms in an integrated voice/data system. We are currently considering the use of this technique to the analysis of an integrated voice/data system with error control schemes.

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