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# 새로운 우선 배분 알고리즘에 관한 분석 - Nonpreemptive 및 Preemptive의 조합 방식

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# Analysis of a New Priority Scheduling Algorithm - Combined Nonpreemptive/Preemptive Discipline

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**ABSTARCT**

*In this paper, we propose a combined nonpreemptive/preemptive priority discipline, and obtain the exact Laplace transform and mean of waiting time for an M/G/1 queueing system with two-priority classes. We also suggest its application to efficient scheduling of heterogeneous messages in the integrated services digital network (ISDN).*

## 1. Introduction

The problems of priority queueing have been studied by many researchers [1], [2]. In the preemptive discipline, an ordinary job for which service is almost completed may be preempted, and, in the nonpreemptive discipline, a priority job may wait even if the ordinary job has just entered service. Obviously, such situations can be avoided by allowing the server to use his discretion to continue or discontinue the service of the ordinary job. In many systems such as a packet-switched system, service time of a job is usually known, and thus the server can use his discretion depending on the remaining service time of the ordinary job.

In this paper, we propose a combined nonpreemptive/preemptive priority discipline. When a priority job arrives at the system being in service of an ordinary job, it waits in queue until the completion of service if the remaining service time of the ordinary job is less than or equal to a fixed discretion time. Otherwise, it will be served immediately, and the ordinary job will go back to the head of the queue of its class. The preemptive rule to be considered in this work is repeat without resampling [1].

## 2. Modeling and Analysis

Consider a single server system with $m$ priority classes which have an infinite buffer. We use the combined nonpreemptive/preemptive repeat without resampling priority discipline between classes and the first-come-first-serve discipline within each class. We assume that class $h$ has priority over class $l$ with discretion time $t_d$, providing that $h < l$. For convenience of analysis, we consider a particular class $k$, $1 \leq k \leq m$, and group other remaining classes into two composite classes, $\alpha$ and $\beta$, i.e., $\alpha \in \{1, ..., k-1\}$ and $\beta \in \{k+1, ..., m\}$. We assume that the class $k$ arrives according to the Poisson process with the rate of $\lambda_k$, and has service time $P_k$ with a general distribution of $S_k(p)$. Before the analysis of waiting time, we define random variables for the class $k$ as the following [1]:

- $W_k$ Waiting time elapsed between the arrival time of a job at the system and the time it first receives service
- $P_{wk}$ Wasted service time elapsed from the time a job receives service until the instant it is preempted
- $P_{sk}$ Successful service time during which service for a job is completed without preemption
- $U_k$ Gross service time that a job actually spends in the server

$D_k$ Breakdown time elapsed from when a preemption occurs in service of a class-$k$ job until the system becomes empty of the class-$\alpha$ jobs

$B_k$ Blocking time elapsed from the instant that a job first receives service until the server is free to serve the next job of the same class.

Now, we analyze the system performance in the steady state without saturation. We define three types of mutually exclusive busy cycles as follows. A type-$\alpha$ cycle commences with the arrival of a class-$\alpha$ job at an idle system, a type-$k$ cycle commences with the arrival of a class-$k$ job at an idle system, and a type-$\beta$ cycle commences with the admission of a class-$\beta$ job into the server. Each cycle ends when the initiator job departs and the system becomes empty of the class-$k$ and -$\alpha$ jobs. Also, we assume that the system is in type-0 cycle when it is idle. Let $\pi_j$ be the steady-state probability of the system being in the type-$j$ cycle, $j \in \{0, \alpha, k, \beta\}$. In analysis of $\pi_j$, it can be stated that the gross service time plays the same role as the service time in case of the nonpreemptive rule. Thus, using the results for the nonpreemptive case, we easily obtain [1], [2]

$$\begin{aligned} \pi_0 &= 1 - \rho, \\ \pi_\alpha &= \rho_\alpha(1 - \rho)/(1 - \rho_k - \rho_\alpha), \\ \pi_k &= \rho_k(1 - \rho)/(1 - \rho_k - \rho_\alpha), \\ \pi_\beta &= \rho_\beta/(1 - \rho_k - \rho_\alpha) \end{aligned} \tag{1}$$

where $\rho_i$ is the system utilization factor of the class $i$, $i \in \{\alpha, k, \beta\}$, i.e., $\rho_i = \lambda_i E[U_i]$, $E[\cdot]$ being expectation, and $\rho = \rho_\alpha + \rho_k + \rho_\beta$. Then, the Laplace transform associated with $W_k$ can be represented by

$$W_k^*(s) = \pi_0 + \sum_{j \in \alpha, k, \beta} \pi_j W_{k|j}^*(s) \quad \text{for the class } k \tag{2}$$

where $W_k^*(s) \triangleq E[e^{-sW_k}]$, and $W_{k|j}$ is the conditional waiting time of a class-$k$ job which arrives at the system in the type-$j$ cycle.

Using the basic equations (1) and (2), let us find waiting time for the system with two-priority classes. For this purpose, we first derive $U_k$ and $B_k$ for $k = 1, 2$. Since there is no priority class higher than 1, it follows that $U_1 = B_1 = P_1$. Defining $N_p$ as the number of preemptions suffered by the class-2 job before it is completed, we easily obtain the probability of having $n$ preemptions as

$$P[N_p = n \mid P_2 = p] = (1 - e^{-\lambda_1[p - t_d]^+})^n e^{-\lambda_1[p - t_d]^+} \tag{3}$$

where $[\cdot]^+$ denotes nonnegative value. Also, we obtain the conditional Laplace transform associated with $P_{w2}$ for $p > t_d$ as [1]

$$E[e^{-sP_{w2}} \mid P_2 = p] = \frac{\lambda_1}{1 - e^{-\lambda_1(p - t_d)}} \int_0^{p - t_d} e^{-(s + \lambda_1)y}\, dy. \tag{4}$$

Letting $Y_1$ be a busy period initiated by the class-1 job, we have a familiar result as [1]

$$Y_1^*(s) = P_1^*(s + \lambda_1 - \lambda_1 Y_1^*(s)). \tag{5}$$

Fig. 1 shows the type-$\beta$ cycle for the class 1 which clearly equals $B_2$.

As shown in Fig. 1, given that $P_2 = p$, $B_2$ consists of $N_p$ dependent pairs, $P_{w2} + D_2$, and one $P_{s2}$ followed by $n$ $Y_1$'s, where $n$ is the number of the class-1 jobs arrived in the nonpreemptable interval $[p, t_d]^-$.[1] Thus, using (3), (4) and the fact that $P_{s2}$ is equal to $p$, we have [1]

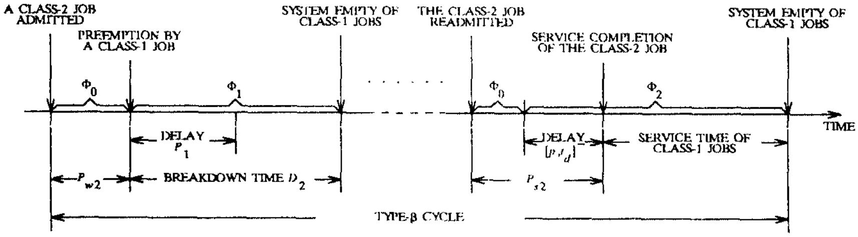


그림 1 Class 1에 대한 베타형 사이클의 재분할.

Fig. 1 Subdivision of the type-$\beta$ cycle for the class 1 ($P_2 = p$).

$$B_2^*(s) = \int_0^\infty \frac{(s+\lambda_1)e^{\lambda_1[p,t_d]^- Y_1^*(s)} e^{-(s+\lambda_1)p}}{s+\lambda_1-\lambda_1 D_2^*(s)(1-e^{-(s+\lambda_1)[p-t_d]^+})} dS_2(p) \tag{6}$$

where $D_2$ is equivalent to $Y_1$. Since $U_2$ consists of only $N_p$ dependent $P_{w2}$'s and one $P_{s2}$, we can directly obtain $U_2^*(s)$ if we replace $D_2^*(s)$ and $Y_1^*(s)$ by 1 in (6).

Now, to derive $W_k^*(s)$, $k = 1, 2$, it remains only to find $W_{k|j}^*(s)$, $j \in \{\alpha, k, \beta\}$. However, it is not easy to obtain $W_{1|\beta}^*(s)$ directly. Thus, we subdivide the type-$\beta$ cycle for the class 1 into three types of cycle $\Phi_i$, $i = \{0, 1, 2\}$, as shown in Fig. 1. We define $\phi_i(p)$ as the conditional probability that a class-1 job arrives in $\Phi_i$ with $P_2 = p$ while the system is in the type-$\beta$ cycle. Since the cycle length of the type $\beta$ for the class 1 is equal to $B_2$, using (3)-(6), we obtain $\phi_i(p)$ as

$$\phi_0(p) = (E[N_p \mid P_2 = p]E[P_{w2} \mid P_2 = p] + [p - t_d]^+)/E[B_2],$$
$$\phi_1(p) = E[N_p \mid P_2 = p]E[D_2]/E[B_2],$$
$$\phi_2(p) = [p, t_d]^-/(1 - \rho_1)E[B_2]. \tag{7}$$

A class-1 job arrived in $\Phi_0$ experiences no waiting delay. Thus, we can represent $W_{1|\beta}^*(s)$ as

$$W_{1|\beta}^*(s) = {}_0[\phi_0(p) + \phi_1(p)W_{1|\Phi_1}^*(s) + \phi_2(p)W_{1|\Phi_2}^*(s)]dS_2(p). \tag{8}$$

Then, $W_{k|j}$, $k|j \in \{1|1, 1|\Phi_1, 1|\Phi_2, 2|\alpha, 2|2\}$, may be regarded as the waiting time in an M/G/1 model where a job arrives in a delay cycle consisting of delay and delay busy period [1], and $B_k$ plays the role of the job service time. Hence, letting the delay of a delay cycle, type-$j$, be $T_{k|j}$, we have

$$W_{k|j}^*(s) = \frac{(1 - \lambda_k E[B_k])(1 - T_{k|j}^*(s))}{E[T_{k|j}](s - \lambda_k + \lambda_k B_k^*(s))}. \tag{9}$$

In (9), we easily obtain the delay, $T_{k|j}$, as

$$T_{1|1} = P_1, \quad T_{1|\Phi_1} = P_1, \quad T_{1|\Phi_2} = [p, t_d]^-,$$
$$T_{2|\alpha} = Y_1, \quad T_{2|2} = B_2. \tag{10}$$

Consequently, by substituting (10) into (9) and using (2), (8), we can find $W_1^*(s)$ and $W_2^*(s)$. Finally, from the $W_1^*(s)$ and $W_2^*(s)$, we can obtain the mean waiting time as

$$E[W_1] = \frac{1}{2(1-\rho_1)}(\lambda_1 E[P_1^2] + \lambda_2 \int_0^\infty [p^2, t_d{}^2]^- dS_2(p)),$$
$$E[W_2] = \frac{1}{2(1-\rho_1)^2}\lambda_1 E[P_1^2] + \frac{1-\rho_1}{2(1-\rho)}\lambda_2 E[B_2^2]. \tag{11}$$

### 3. Numerical Example

We have proposed a combined nonpreemptive/preemptive priority discipline, and analyzed its performance. Note that, when $t_d$ is zero, it coincides with the preemptive discipline, and when $t_d$ is infinite, it is identical to the nonpreemptive discipline. Fig. 2 shows a numerical example of the mean waiting time $E[W_1]$ and $E[W_2]$ versus arrival rate $\lambda_1$ for different values of $t_d$ when the service time of the classes 1 and 2 is constant.

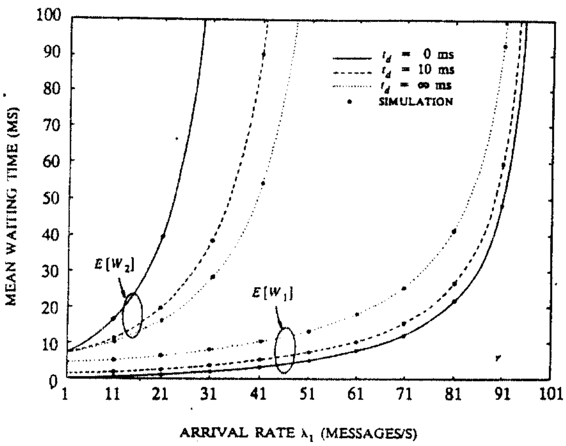


그림 2 $t_d$ = 0, 10, ∞ ms 일때 class 1과 class 2의 arrival rate $\lambda_1$에 대한 평균 waiting 시간.

Fig. 2 Mean waiting time of the classes 1 and 2 versus arrival rate $\lambda_1$ for $t_d$ = 0, 10, ∞ ms. $P_1$ = 10 ms, $P_2$ = 20 ms and $\lambda_2$ = 20 messages/s.

One can see from Fig. 2 that the mean waiting time of the combined discipline has the following relations with those of the nonpreemptive and preemptive disciplines: $E[W_1]|_{preempt.} \le E[W_1]|_{comb.} \le E[W_1]|_{nonpreempt.}$ and $E[W_2]|_{preempt.} \ge E[W_2]|_{comb.} \ge E[W_2]|_{nonpreempt.}$ . The discretion time $t_d$ can be chosen so as to optimize a given objective function. For example, suppose that we want to minimize the total delay cost of the classes 1 and 2. Assuming the cost to be linear with the delay costs per unit time, $K_1$ and $K_2$, for the

[1] $[\cdot, \cdot]^-$ denotes the minimum value of the two values inside the bracket.

classes 1 and 2, respectively, the cost function can be represented by $K_1\lambda_1E[W_1] + K_2\lambda_2E[W_2]$. Then, we can find the optimal discretion time that minimizes the cost function. Also, the combined discipline has the advantage that the waiting delay of the class 1 effected by the class 2 is restricted within $t_d$. From this property, we may mitigate the constraint on the message size of the class 2 which, in the nonpreemptive case, is imposed to meet the delay requirement of the class 1. Therefore, with the proper discretion time $t_d$, we can schedule heterogeneous messages efficiently under the constraints of their delay requirements. This aspect is under study in the ISDN environment.

**References**

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