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Estimation Algorithms of the Model Parameters of Robotic Manipulators

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Abstract

The dynamic equations of robotic manipulators can be derived from either Newton-Euler equation or Lagrangian equation. Model parameters which appear in the resulting dynamic equation are the nonlinear funtions of both the inertial parameters and the geometric parameters of robotic manipulators. The identification of the model parameters is important for advanced robot control. In the previous methods for the identification of the model parameters, the geometric parameters are required to be predetermined, or the robotic manipulators are required to follow some special motions. In this paper, we propose an approach to the identification of the model parameters, in which prior knowledge of the geometric parameters is not necessary. We show that the estimation equation for the model parameters can be formulated in an upper block triangular form. Utilizing the special structures, we obtain a simplified least-square estimation algorithm for the model parameter identification. To illustrate the practical use of our method, a 4DOF SCARA robot is examined.

1. Introduction

Dynamic modelling of a given plant is important for the control system design. Dynamic equations of robotic manipulators with serial links can be obtained by the Newton-Euler method [18] or the Lagrangian-Euler method [15,22]. Efficient programs for the symbolic generation of the dynamic equations of the manipulators are now available [8,13,16]. Various coefficients appear in the resultant dynamic equations of robotic manipulators. Identification of these coefficients is important for advanced robot control. In fact, many advanced control schemes for robotic manipulator which have been presented in the recent literature need the information on either all or some of these coefficients [4,7,11,14,17,19,20,21,23]. These coefficients, which will be called model parameters from now on, are the nonlinear functions of the geometric parameters (the constant parameters of the homogeneous transformation matrices) and the inertial parameters (mass, center of mass, and moments of inertia of links). In this paper, we consider the identification problem of these model parameters.

Most of the prior work addressed the identification problem of the inertial parameters under the condition that geometric parameters are priorly known. In fact, the geometric parameters can be seperately identified through some calibration procedure [5]. Unfortunately, these methods fail to identify some of the inertial parameters because each robotic manipulator has its own degree of freedom and hence the effects of some inertial parameters on robot motions are hidden. In [3], the unidentifiable inertial parameters are characterized. In [2], all inertial parameters could be seperately identified by disassemblying robotic manipulators into components and by applying a two-wire suspension method. However, the identification procedure involves much labor. Furthermore, some parts of manipulators are difficult to be disassembled.

The prior work closely related to our work are [3,9,24]. The estimation methods proposed in [3,24] can be used to identify the model parameters when the geometric parameters are pre-known. In [3] some experimental results were also presented. In [9], the geometric parameters need not to be known in advance. However, manipulators are requried to follow various predetermined motions. The success of the approach depends highly on how closely the manipulators can follow the predetermined motions.

Our estimation algorithm for the model parameters requires neither the pre-knowledge of the geometric parameters nor any special motions. It was shown in [3] that the estimation model for the inertial parameters has an upper block triangular form. We show that the estimation model for the model parameters also has an upper block triangular form. We show that this special structure can be utilized to get computationally simple estimation algorithm for the model parameters. Some simulation results for the case of 4DOF SCARA robot are presented to illustrate the practical use of our estimation method.

Finally, we introduce some notations needed in section 2 and 3. For a function $f : R^p \rightarrow R^r$, $D_j f(x)$ denotes the first partial derivative of f at $x \in R^p$ with respect to the jth argument. A column vector x with scalar components x_i, i = 1,...,p is denoted by $x \triangleq (x_1, \ldots, x_p)$. A row vector x with scalar components x_i, i = 1,...,p is denoted by $x \triangleq [x_1 \ldots x_p]$. The tranpose of a

vector x is denoted by x^T. The functions $f_i : R^p \to R^r$, i=1,...n. are linearly dependent on R^p if there are constants α_i, i = 1,...,n, (not all zero) such that $\sum_{i=1}^{n} \alpha_i f_i(x) \equiv 0,\ x \in R^p$.
The functions $f_i : R^p \to R^r$, i = 1,...,n are linearly dependent at each $x \in R^p$ if, for each $x \in R^p$, there are constants $\alpha_i(x)$, i = 1,...,n, (not all zero) such that $\sum_{i=1}^{n} \alpha_i(x) f_i(x) \equiv 0$.
It should be clear from these definitions that the functions which are linearly independent at each point are not necessarily linearly independent on the whole domain.

2. Main Result

The Lagrangian formulation of the dynamic equations of a system with n degree of freedom is given [6] by

$$\left[\frac{d}{dt} D_2 L(q,\dot{q}) - D_1 L(q,\dot{q})\right]^T = \tau, \qquad (2.1)$$

where,
- $q(t) \in R^n$: the generalized coordinate vector of the system,
- $\dot{q}(t) \in R^n$: the time derivative of q(t),
- $\tau(t) \in R^n$: the generalized force (or torque) vector applied to the system,
- $L(q,\dot{q})$: the Lagrangian function of the system (= kinetic energy - potential energy).

The Lagrangian function L of a robotic manipulators with serial links can be written [22] as

$$L(q,\dot{q}) = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i} \mathrm{Trace}\left(\frac{\partial T_i}{\partial q_j} J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T\right)\dot{q}_j \dot{q}_k + \sum_{i=1}^{n} m_i \bar{g}^T T_i \bar{r}_i, \qquad (2.2)$$

where,
- $T_i \in R^{4\times4}$: the homogeneous transformation matrix relating the coordinate frame of the ith link to that of the 0th link (the base coordinate frame),
- $J_i \in R^{4\times4}$: the pseudo inertia matrix of the ith link,
- $\bar{g} \triangleq [g_x\ g_y\ g_z\ 0]$: the gravity vector with respect to the base coordinate frame,
- $\bar{r}_i \triangleq [\bar{x}_i\ \bar{y}_i\ \bar{z}_i\ 1]$: the position vector of the center of the ith link mass with respect to the ith coordinate frame,
- m_i : the mass of the ith link.

Here, the inertial parameters are $m_i, m\bar{x}_i, m\bar{y}_i, m\bar{z}, I_{ixx}, I_{iyy}, I_{izz}, I_{ixy}, I_{iyz}, I_{ixz}$, i = 1, ...,n. Hence, the total number of the inertial parameters of a manipulator with n DOF is 10n. For each i = 1,...,n, let

$$L_i(q,\dot{q}) \equiv \tfrac{1}{2}\sum_{j=1}^{i}\sum_{k=1}^{i} \mathrm{Trace}\left(\frac{\partial T_i}{\partial q_j} J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T\right)\dot{q}_j \dot{q}_k + m_i \bar{g}^T T_i \bar{r}_i \qquad (2.3)$$

Then, the Lagrangian function L in (2.2) can be written as

$$L(q,\dot{q}) = \sum_{i=1}^{n} L_i(q,\dot{q}) \qquad (2.4)$$

For the notational convenience we assume that all joint are rotational. However, it will be clear from the developments that such an assumption entails no loss of generality. We define the geometric parameters d_i, a_i, α_i of the ith link as in Fig.1. Hence, the total number of the geometric parameters of a manipulators with n DOF is 3n. For notational convenience, we write $V_i \triangleq (m_i, m\bar{x}_i, m\bar{y}_i, m\bar{z}_i, I_{ixx}, I_{iyy}, I_{izz}, I_{ixy}, I_{iyz}, I_{ixz})$, $W_i \triangleq (d_i, a_i, \alpha_i)$, i =1,...,n. Let $V \triangleq (V_1,...,V_n)$ and $W \triangleq (W_1,...,W_n)$.

The homogeneous transformation matrix A_i relating the coordinate frame of the ith link to that of the (i-1)th link is given [22] by

$$A_i = \hat{A}_i C_i, \qquad (2.5)$$

where,

$$\hat{A}_i \triangleq \mathrm{Rot}(z_{i-1}, q_i), \qquad (2.6)$$

$$C_i \triangleq \mathrm{Trans}(0,0,d_i)\mathrm{Trans}(a_i,0,0)\mathrm{Rot}(x_i, \alpha_i) \qquad (2.7)$$

Namely, each A_i can be factorized into two matrices such that $\hat{A}_i$ depends only on q_i and C_i depends only on the geometric parameters d_i, a_i, α_i of the ith link. By (2.5), T_i is then written as

$$T_i = A_1 A_2 \cdots A_i = \hat{A}_1 C_1 \hat{A}_2 C_2 \cdots \hat{A}_i C_i, \quad i=1,...,n \qquad (2.8)$$

Therefore, we can always find an integer p_i and matrix functions $\hat{T}_i : R^i \to R^{4\times p_i}$ and $\hat{C}_i : R^{3i} \to R^{p_i\times4}$ so that T_i can be factorized in the form :

$$T_i = \hat{T}_i(q_1,...,q_i)\hat{C}_i(W_1,...,W_i) \qquad (2.9)$$

L_i in (2.3) can be written as

$$L_i(q,\dot{q}) = \tfrac{1}{2}\sum_{j=1}^{i}\sum_{k=1}^{i} \dot{q}_j \dot{q}_k \mathrm{Trace}\left(\frac{\partial \hat{T}_i}{\partial q_j} \hat{J}_i \left(\frac{\partial \hat{T}_i}{\partial q_k}\right)^T\right) + \bar{g}^T \hat{T}_i \hat{r}_i, \qquad (2.10)$$

where, $\hat{J}_i \triangleq \hat{C}_i J_i \hat{C}_i^T$ and $\hat{r}_i \triangleq m_i \hat{C}_i \bar{r}_i \qquad (2.11)$

From (2.9), (2.10) and (2.11), we see that (1) L_i does not depend on $q_k, \dot{q}_k$, k = i+1,...,n. and (2) L_i is linear with respect to $\hat{J}_i$ and $\hat{r}_i$. By

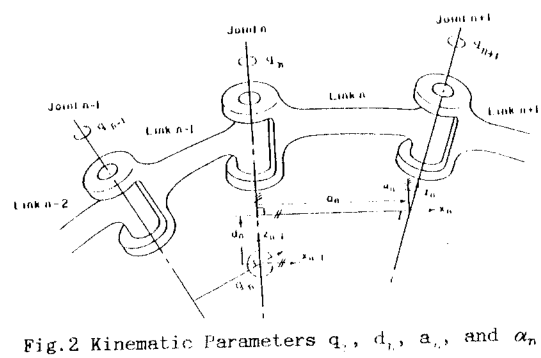

Fig.2 Kinematic Parameters q_n, d_n, a_n, and α_n

these facts, it is always possible to find an integer n_i, functions $F_{ij} : R^{2i} \rightarrow R$ and $X_{ij} : R^{3i+10} \rightarrow R$, $j = 1,\ldots,n_i$ such that L_i can be formulated in the form :

$$L_i(q,\dot{q}) = \sum_{j=1}^{n_i} F_{ij}(q_1,\ldots,q_i,\dot{q}_1,\ldots,\dot{q}_i)X_{ij}(V_i,W_1,\ldots,W_i) \tag{2.12}$$

In particular, the X_{ij} are linear with respect to V_i. Consequently, L in (2.4) can be written as

$$L(q,\dot{q}) = \sum_{i=1}^{n} \sum_{j=1}^{n_i} F_{ij}(q_1,\ldots,q_i,\dot{q}_1,\ldots,\dot{q}_i)X_{ij}(V_i,W_1,\ldots,W_i) \tag{2.13}$$

Next, we rearrange the terms in (2.13) as follows. For each $i = 1,\ldots,n$, define $\hat{L}_i$ by the sum of all terms in (2.13) that depend on at least one of q_i, $\dot{q}_i$ but not on any of q_k, $\dot{q}_k$, $k = i+1,\ldots,n$. Let m_i be the total number of the terms in $\hat{L}_i$. We denote the terms in $\hat{L}_i$ by $\hat{F}_{ij}\hat{X}_{ij}$. Then,

$$L(q,\dot{q}) = \sum_{i=1}^{n} \hat{L}_i = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \{\hat{F}_{ij}(q_1,\ldots,q_i,\dot{q}_1,\ldots,\dot{q}_i)\cdot \hat{X}_{ij}(V_i,\ldots,V_n,W_1,\ldots,W_n)\} \tag{2.14}$$

The representation of each $\hat{L}_i$ can be further simplified. If there is a term $\hat{F}_{ij}\hat{X}_{ij}$ in $\hat{L}_i$ such that for some constants α_{ik} (not all zero), $\hat{F}_{ij} = \sum_{k \neq j} \alpha_{ik}\hat{F}_{ik}$, the term $\hat{F}_{ij}\hat{X}_{ij}$ is removed from $\hat{L}_i$ while the $\hat{F}_{ik}\hat{X}_{ik}$ are replaced by $\hat{F}_{ik}(\hat{X}_{ik} + \alpha_{ik}\hat{X}_{ij})$, respectively. If there is a term $\hat{F}_{ij}\hat{X}_{ij}$ in $\hat{L}_i$ such that for some constants β_{ik} (not all zero), $\hat{X}_{ij} = \sum_{k \neq j} \beta_{ik}\hat{X}_{ik}$, the term $\hat{F}_{ij}\hat{X}_{ij}$ is removed from $\hat{L}_i$ while the $\hat{F}_{ik}\hat{X}_{ik}$ are replaced by $(\hat{F}_{ik} + \beta_{ik}\hat{F}_{ij})\hat{X}_{ik}$, respectively. We assume that each $\hat{L}_i$ in (2.14) has been simplified through the above procedure. Then, we call $\hat{X}_{ij}$ in (2.14) the model parameters. Then, $m \triangleq \sum_{i=1}^{n} m_i$ is the total number of the model parameters. As will be seen later, m does not necessarily represent the minimal number of the model parameters for the determination of the dynamic equation of a robotic manipulator. Let $\hat{F}_i \triangleq (\hat{F}_{i1},\ldots,\hat{F}_{im_i})$, $\hat{F} \triangleq (\hat{F}_1,\ldots,\hat{F}_n)$, $\hat{X}_i \triangleq (\hat{X}_{i1},\ldots,\hat{X}_{im_i})$, and $\hat{X} \triangleq (\hat{X}_1,\ldots,\hat{X}_n)$. Then, (2.14) can be rewritten in a simpler form :

$$L(q,\dot{q}) = \sum_{i=1}^{n} \{[\hat{F}_i(q_1,\ldots,q_i,\dot{q}_1,\ldots,\dot{q}_i)]^T \cdot \hat{X}_i(V_i,\ldots,V_n,W_1,\ldots,W_n)\}, \tag{2.15}$$

What remains now is to show

$$m_i \geq 1, \; i = 1,\ldots,n. \tag{2.16}$$

Let i be an integer such that $1 \leq i \leq n$. It can be easily shown that if $\dot{q}_i \neq 0$,

$$a_{ki} \equiv (\dot{q}_i)^2 \mathrm{Trace}\left(\frac{\partial \hat{T}_k}{\partial q_i}\right)\hat{J}_k\left(\frac{\partial \hat{T}_k}{\partial q_i}\right)^T > 0, \quad k = i,\ldots,n. \tag{2.17}$$

This implies that in (2.10), any term containing $(\dot{q}_i)^2$ can not be cancelled out by any other terms. In particular, there are functions $f_i : R^i \rightarrow R$, $x_i : R^{10} \rightarrow R$ such that a_{ii} can be written as

$$a_{ii} = (\dot{q}_i)^2 f_i(q_1,\ldots,q_i)x_i(V_i) \tag{2.18}$$

Due to the special forms of the $\hat{F}_i$ and the $\hat{X}_i$ in (2.14), this implies that $(\hat{F}_i)^T\hat{X}_i$ must contain at least a_{ii}. This proves (2.16). As will be seen soon, rearrangement of the Lagrangian function in the form (2.15) has several advantages in simplifying the estimation method of the model parameters.

Now, we formulate two kinds of identification models for the model parameters. Let $\hat{F} \triangleq (\hat{F}_1,\ldots,\hat{F}_n)$ and $\hat{X} \triangleq (\hat{X}_1,\ldots,\hat{X}_n)$. By (2.15), (2.1) can be written as

$$U(q,\dot{q},\ddot{q})\,\hat{X} = \tau, \tag{2.19}$$

where,

$$U(q,\dot{q},\ddot{q}) = \left[\frac{d}{dt} D_2\hat{F}(q,\dot{q}) - D_1\hat{F}(q,\dot{q})\right]^T \tag{2.20}$$

By the definition of the $\hat{F}_i$, we see that

$$\frac{\partial \hat{F}_i}{\partial q_k} \equiv 0 \text{ and } \frac{\partial \hat{F}_i}{\partial \dot{q}_k} \equiv 0, \; k = i+1,\ldots,n, \; i = 1,\ldots,n \tag{2.21}$$

Consequently, U in (2.19) has the following upper block triangular form :

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ 0 & U_{22} & \cdots & U_{2n} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & U_{nn} \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \vdots \\ \hat{X}_n \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \tag{2.22}$$

where,

$$U_{ij} \triangleq \left[\frac{d}{dt}\left(\frac{\partial \hat{F}_j}{\partial \dot{q}_i}\right) - \left(\frac{\partial \hat{F}_j}{\partial q_i}\right)\right]^T, \quad Y_i = \tau_i. \tag{2.23}$$

When the geometric parameters W_i, $i = 1,\ldots,n$ are known, the $\hat{X}_{ij}$ are reduced to be the linear functions of $V_i,\ldots,V_n$. In other words, there is a upper block triangular matrix M such that $\hat{X} = MV$. This with (2.22) shows that an identification model for the inertial parameters also has a upper block triangular form. Hence, when the geometric parameters are known, our result recovers the one in [3].

Usually $m > n$. From the aspect of computational load, it is desirable to make m as small as possible. We have already simplified $\hat{L}_i$, $i = 1,\ldots,n$ to achieve a small m. However, m can be further reduced by eliminating linearly dependent columns and components from U and $\hat{X}$, respectively. The following simplification method is a simple extension of the one used in [24]. Let $U_i \triangleq (U_{1i}^T,\ldots,U_{ii}^T)^T$, $i = 1,\ldots,n$. Each U_i has m_i column vectors denoted by U_i^k, $k = 1,\ldots,m_i$. Let i be an integer such that $1 \leq i \leq n$. If there is a column vector U_i^j such that for some constants α_{ik} (not all zero) $U_i^j = \sum_{k \neq j} \alpha_{ik} U_i^k$ on R^{3n}, the column U_i^j and the component $\hat{X}_{ij}$ are eliminated from U_i and $\hat{X}_i$, respectively. Then, the other components $\hat{X}_{ik}$, $k \neq j$ in $\hat{X}_i$ are replaced by $\hat{X}_{ik} + \alpha_{ik} \hat{X}_{ij}$, respectively. After all columns of U are made linearly independent on R^{3n}, all linearly dependent components are eliminated from $\hat{X}$ in the following way. Suppose that some components of $\hat{X}$ are linearly dependent on R^{13n}. In this case, there is always a component $\hat{X}_{ij}$ in $\hat{X}$ such that for some constants β_{ik}, $k = j+1, \ldots, m$ and γ_{pk}, $k = 1, \ldots, m_p$, $p = i+1, \ldots, n$, (not all zero),

$$\hat{X}_{ij} = \sum_{k=j+1}^{m_i} \beta_{ik} \hat{X}_{ik} + \sum_{p=i+1}^{n} \sum_{k=1}^{m_p} \gamma_{pk} \hat{X}_{pk}, \tag{2.24}$$

$$V \in R^{10n}, \quad W \in R^{3n}.$$

Then, $\hat{X}_{ij}$ and U_i are eliminated from $\hat{X}_i$ and U_i, respectively. while U_i^k, $k = j+1, \ldots, m_i$ and U_p^k, $k = 1, \ldots, m_p$, $p = i+1, \ldots, n$ are replaced by $U_i^k + \beta_{ik} U_i^j$ and $U_p^k + \gamma_{pk} U_i^j$, respectively. Note that the columns of U still remain linearly independent on R^{3n} after such operations on U and $\hat{X}$. Furthermore, the resulting identification model preserves the upper block triangular form. In fact, this simplification method leads to the minimal number m(in some sense) of the model parameters to be identified to determine the dynamic equation of a given robotic manipulator. We assume that (2.22) represents the identification model obtained by the above simplification procedure.

The $\hat{X}_{ij}$ in (2.22) can be obtained by using the least square method. The required measurement data are the trajectories of $q, \dot{q}, \ddot{q}$, and τ. While $q, \dot{q}$ can be directly measured, $\ddot{q}$, τ should be estimated in an indirect way. Suppose we have N data points. Each data point determines numerically U and τ in (2.19), which will be denoted by $U(i)$, $\tau(i)$, $i = 1, \ldots, N$. Then, $\hat{X}$ can be estimated by

$$\hat{X} = (\hat{U}^T \hat{U})^{-1} \hat{U}^T \hat{Y}, \tag{2.25}$$

where,

$$\hat{U} \triangleq \begin{bmatrix} U(1) \\ \bullet \\ \bullet \\ \bullet \\ U(N) \end{bmatrix} \quad \text{and} \quad \hat{Y} \triangleq \begin{bmatrix} Y(1) \\ \bullet \\ \bullet \\ \bullet \\ Y(N) \end{bmatrix}. \tag{2.26}$$

While the data of τ can be easily estimated from joint motor currents, the data of $\ddot{q}$ are obtained by passing the data of $\dot{q}$ through a band-limited differentiator [3]. When it is desired to avoid the differentiation of $\dot{q}$, the following approach can be taken. Integrating (2.19) from time t_1 to time t_2, we see that (2.22) still holds with

$$U_{ij} \triangleq \left[\left. \frac{\partial \hat{F}_j}{\partial \dot{q}_i} \right|_{t=t_2} - \left. \frac{\partial \hat{F}_j}{\partial \dot{q}_i} \right|_{t=t_1} - \int_{t_1}^{t_2} \left(\frac{\partial \hat{F}_j}{\partial q_i} \right) dt \right]^T,$$

$$Y \triangleq \int_{t_1}^{t_2} \tau \, dt. \tag{2.27}$$

In this case, the data of $\ddot{q}$ are not necessary. Instead, integration of $D_i \hat{F}(q, \dot{q})$ is required. This integration approach was first considered in [3] for load estimation. Choosing N different pairs (t_1, t_2), we can construct $\hat{U}$ and Y required in (2.25).

The inverse of $\hat{U}^T \hat{U} \in R^{m \times m}$ is computationally difficult when m is large. The following recursive algorithm utilizing the special structure of U in (2.22) may be useful for the reduction of computational load.

$$\hat{X}_i = (\hat{U}_{ii}^T \hat{U}_{ii})^{-1} \hat{U}_{ii}^T (\hat{Y}_i - \sum_{j=i+1}^{n} \hat{U}_{ij} \hat{X}_j), \tag{2.28}$$

$$i = n, (n-1), \ldots, 1.$$

where,

$$\hat{U}_{ij} \triangleq \begin{bmatrix} U_{ij}(1) \\ \bullet \\ \bullet \\ \bullet \\ U_{ij}(N) \end{bmatrix},$$

$$\hat{Y}_i \triangleq \begin{bmatrix} Y_i(1) \\ Y_i(2) \\ \bullet \\ \bullet \\ \bullet \\ Y_i(N) \end{bmatrix} \qquad \begin{matrix} i = 1, \ldots, n \\ j = i, \ldots, n \end{matrix} \tag{2.29}$$

In other words, the $\hat{X}_i$, $i = 1, \ldots, n$ are estimated each by each in the reversed order. In (2.28), only the inverses of $\hat{U}_{ii}^T \hat{U}_{ii} \in R^{m_i \times m_i}$ need to be computed.

The selection of data points is important for the success of these algorithms. In both algorithms (2.25) and (2.28), the condition :

$$\text{Rank } \hat{U}_{ii} = m_i, \quad i = 1, \ldots, n. \tag{2.30}$$

is necessary and sufficient for the existence of the required inverses. If (2.30) is satisfied, and no measurement and computational errors are involved, these algorithms produce the true value of the model parameters.

3. An Example

The 4DOF SCARA Robot shown in Fig.2 has 4 links. The first, second, and fourth links are rotational while the third link is translational. The first, third, and forth links are symmetric about the x axis of the joint coordinate frame while the second link is not symmetric about the x axis of the joint coordinate frame. The kinematic parameters are given in Table.1.

Table.1 Kinematic parameters of the robot in Fig.2

link	α_i	a_i	d_i	q_i
1	0	a_1	0	q_1
2	0	a_2	0	q_2
3	0	0	0	q_3
4	0	0	0	q_4

And the pseudo inertia matrix J_i is given by

$$J_i=\begin{bmatrix} \frac{-I_{ixx}+I_{iyy}+I_{izz}}{2} & I_{ixy} & I_{ixz} & m_i\bar{x}_i \\ I_{ixy} & \frac{I_{ixx}-I_{iyy}+I_{izz}}{2} & I_{iyz} & m_i\bar{y}_i \\ I_{ixz} & I_{iyz} & \frac{I_{ixx}+I_{iyy}-I_{izz}}{2} & m_i\bar{z}_i \\ m_i\bar{x}_i & m_i\bar{y}_i & m_i\bar{z}_i & m_i \end{bmatrix} \tag{3.1}$$

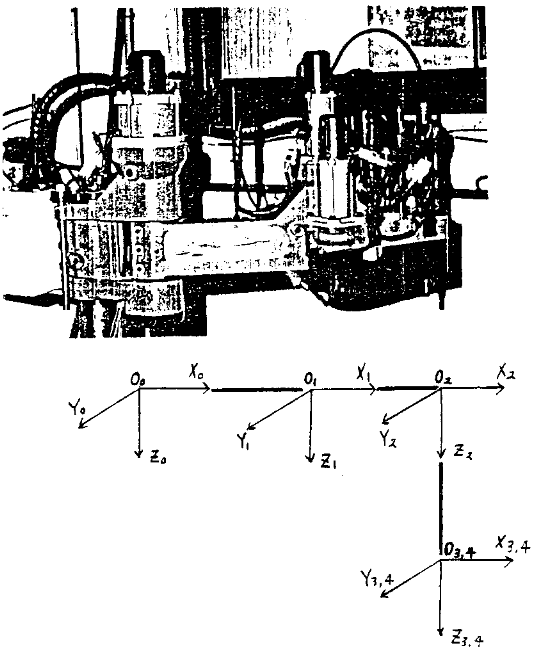

Fig.2 4DOF SCARA Robot and its coordinate frames

Let $S_i \triangleq \sin q_i$, $C_i \triangleq \cos q_i$, $i = 1,\dots,4$, and $g = 9.8(\text{m/sec}^2)$ Then, L_i, $i = 1,\dots,4$ in (2.3) are given as follows.

$$L_1(q,\dot q) = \tfrac{1}{2}\dot q_1^2\,(I_{1zz}+m_1a_1^2) \tag{3.2}$$

$$\begin{aligned} L_2(q,\dot q) = {} & \tfrac{1}{2}\dot q_1^2\,(I_{2zz}+m_2a_1^2+m_2a_2^2+2m_2\bar x_2a_2) \\ & + C_2\dot q_1^2\,(m_2a_1a_2+m_2\bar x_2a_1) - S_2\dot q_1^2\,m_2\bar y_2a_1 \\ & + \dot q_1\dot q_2(I_{2zz}+m_2a_2^2+2m_2\bar x_2a_2) \\ & + C_2\dot q_1\dot q_2(m_2a_1a_2+m_2\bar x_2a_1) - S_2\dot q_1\dot q_2m_2\bar y_2a_1 \\ & + \tfrac{1}{2}\dot q_2^2\,(I_{2zz}+m_2a_2^2+2m_2\bar x_2a_2) \end{aligned} \tag{3.3}$$

$$\begin{aligned} L_3(q,\dot q) = {} & \tfrac{1}{2}\dot q_1^2\,(I_{3zz}+m_3a_1^2+m_3a_2^2) \\ & + C_2\dot q_1^2\,m_3a_1a_2 + \tfrac{1}{2}\dot q_2^2\,(I_{3zz}+m_3a_2^2) \\ & + \tfrac{1}{2}\dot q_3^2\,m_3 + q_3m_3g + \dot q_1\dot q_2(I_{3zz}+m_3a_2^2) \\ & + C_2\dot q_1\dot q_2m_3a_1a_2 \end{aligned} \tag{3.4}$$

$$\begin{aligned} L_4(q,\dot q) = {} & \tfrac{1}{2}\dot q_1^2\,(I_{3zz}+m_3a_1^2+m_3a_2^2) \\ & + C_2\dot q_1^2\,m_3a_1a_2 + \tfrac{1}{2}\dot q_2^2\,(I_{4zz}+m_4a_2^2) \\ & + \tfrac{1}{2}\dot q_3^2\,m_4 + q_3m_4g + \dot q_1\dot q_2(I_{4zz}+m_4a_2^2) \\ & + C_2\dot q_1\dot q_2m_4a_1a_2 + \tfrac{1}{2}\dot q_4^2\,I_{4zz} + \dot q_1\dot q_4I_{4zz} \\ & + \dot q_2\dot q_4I_{4zz} \end{aligned} \tag{3.5}$$

By the regrouping procedure suggested in section 2, we obtain

$$\begin{aligned} \hat L_1 = {} & \tfrac{1}{2}\dot q_1^2\,(I_{1zz}+m_1a_1^2+I_{2zz}+m_2a_1^2+m_2a_2^2 \\ & + 2m_2\bar x_2a_2 + I_{3zz}+m_3a_1^2+m_3a_2^2+I_{4zz} \\ & + m_4a_1^2+m_4a_2^2) \end{aligned} \tag{3.6}$$

$$\begin{aligned} \hat L_2 = {} & (\tfrac{1}{2}\dot q_2^2+\dot q_1\dot q_2)(I_{2zz}+m_2a_2^2+2m_2\bar x_2a_2+I_{3zz} \\ & + m_3a_2^2+I_{4zz}+m_4a_2^2) + (\dot q_1^2C_2+\dot q_1\dot q_2C_2)\bullet \\ & (m_2a_1a_2+m_2\bar x_2a_1+m_3a_1a_2+m_4a_1a_2) \\ & - (\dot q_1^2S_2+\dot q_1\dot q_2S_2)m_2\bar y_2a_1 \end{aligned} \tag{3.7}$$

$$\hat L_3 = (\tfrac{1}{2}\dot q_3^2+q_3g)(m_3+m_4) \tag{3.8}$$

$$\hat L_4 = (\tfrac{1}{2}\dot q_4^2+\dot q_1\dot q_4+\dot q_2\dot q_4)I_{4zz} \tag{3.9}$$

Note from (3.6) – (3.9) that

$$m_1 = m_3 = m_4 = 1,\ m_2 = 3,\ m = 6 \tag{3.10}$$

In this example, the simplification procedure indicated in section 2 does not help to reduce the m further. Hence, the minal number of the model parameters to be identified is $m = 6$ and the vector $\hat X$ of the model parameters is given by

$$\hat X \triangleq (\hat X_1,\ \hat X_2,\ \hat X_3,\ \hat X_4) \tag{3.11}$$

where,

$$\begin{aligned} \hat X_1 = {} & \hat X_{11} = I_{1zz}+m_1a_1^2+I_{2zz}+m_2a_1^2+m_2a_2^2 \\ & + 2\cdot m_2\bar x_2a_2 + I_{3zz}+m_3a_1^2+m_3a_2^2+I_{4zz} \\ & + m_4a_1^2+m_4a_2^2 \\ \hat X_2 = {} & (\hat X_{21},\ \hat X_{22},\ \hat X_{23}) = (I_{2zz}+m_2a_2^2+2m_2\bar x_2a_2 \\ & + I_{3zz}+m_3a_2^2+I_{4zz}+m_4a_2^2,\ m_2a_1a_2+m_2\bar x_2a_1 \\ & + m_3a_1a_2+m_4a_1a_2,\ m_2\bar y_2a_1) \\ \hat X_3 = {} & \hat X_{31} = m_3+m_4 \\ \hat X_4 = {} & \hat X_{41} = I_{4zz} \end{aligned} \tag{3.12}$$

The identification models for these model parameters is given by

$$\begin{bmatrix} U_{11} & U_{12} & 0 & U_{14} \\ 0 & U_{22} & 0 & U_{24} \\ 0 & 0 & U_{33} & 0 \\ 0 & 0 & 0 & U_{44} \end{bmatrix} \begin{bmatrix} \hat X_1 \\ \hat X_2 \\ \hat X_3 \\ \hat X_4 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} \tag{3.13}$$

where, for the case of the first identification model,

$$\begin{aligned} & U_{11} = \ddot q_1,\ U_{14} = U_{24} = \ddot q_4,\ U_{33} = \ddot q_3 - g, \\ & U_{44} = \ddot q_1+\ddot q_2+\ddot q_4 \end{aligned} \tag{3.14}$$

$$\begin{aligned} U_{12} = [\ & \ddot q_2\quad \{-2\dot q_1\dot q_2S_2-\dot q_2^2S_2+(2\ddot q_1+\ddot q_2)C_2\} \\ & \{-2\dot q_1\dot q_2C_2-\dot q_2^2C_2-(2\ddot q_1+\ddot q_2)S_2\}\ ] \end{aligned} \tag{3.15}$$

$$U_{22} = [\ \ddot q_1+\ddot q_2\quad \ddot q_1C_2+\dot q_1^2S_2\quad -\ddot q_1S_2+\dot q_1^2C_2\ ] \tag{3.16}$$

and for the case of the second identification model,

$$\begin{aligned} & U_{11} = \dot q\,|\,t_2 - \dot q_1\,|\,t_1,\ U_{14} = U_{24} = \dot q_4\,|\,t_2 - \dot q_4\,|\,t_1, \\ & U_{33} = \dot q_3\,|\,t_2 - \dot q_3\,|\,t_1 - g(t_2-t_1), \\ & U_{44} = \{\dot q_1+\dot q_2+\dot q_4\}\,|\,t_2 - \{\dot q_1+\dot q_2+\dot q_4\}\,|\,t_1 \end{aligned} \tag{3.14$'$}$$

$$\begin{aligned} U_{12} = {} & [\ \dot q_2\quad (2\dot q_1+\dot q_2)C_2\quad -(2\dot q_1+\dot q_2)S_2\ ]\,|\,t_2 \\ & - [\ \dot q_2\quad (2\dot q_1+\dot q_2)C_2\quad -(2\dot q_1+\dot q_2)S_2\ ]\,|\,t_1 \end{aligned} \tag{3.15$'$}$$

$$\begin{aligned} U_{22} = {} & [\ \dot q_1+\dot q_2\quad \dot q_1C_2\quad -\dot q_1S_2\ ]\,|\,t_2 \\ & - [\ \dot q_1+\dot q_2\quad \dot q_1C_2\quad -\dot q_1S_2\ ]\,|\,t_1 \\ & + [\ 0\quad \int_{t_1}^{t_2}(\dot q_1\dot q_2+\dot q_1^2)S_2dt\quad \int_{t_1}^{t_2}(\dot q_1\dot q_2+\dot q_1^2)C_2dt \end{aligned} \tag{3.16$'$}$$

The sample data of q, $\dot q$, τ used for the identification of the model parameters are shown in Fig.3 – Fig.5, where ⓘ, $i = 1,\dots,4$ indicate the ith joint. The true values of the model parameters are assumed to be given as in Table.2. The simulation results in Table.2 show that both of the two estimation algorithms proposed in Section 2 work well for this example.

4.Conclusion

We have proposed two identification models for the model parameters and two alternative estimation algorithms. Advantages and disadvantages of these methods should be further investigated. For an instance, the possible measurement errors and computational errors should be taken into account. It is an important problem how to select the data points so that the rank condition (2.30) is satisfied.

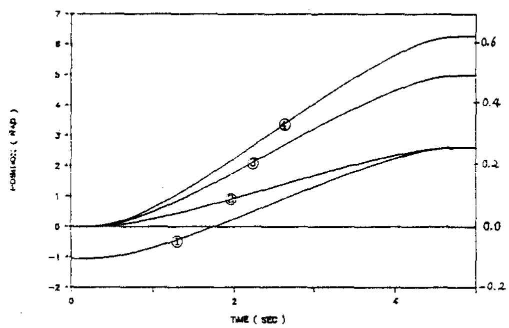

Fig.3 Position Trajectory

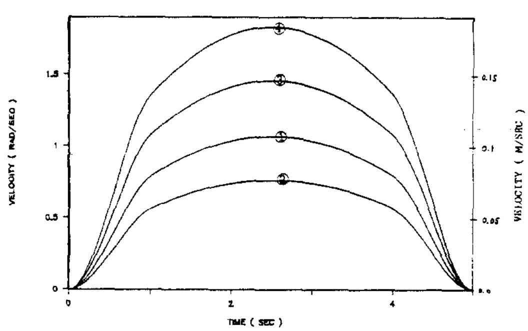

Fig.4 Velocity Trajectory

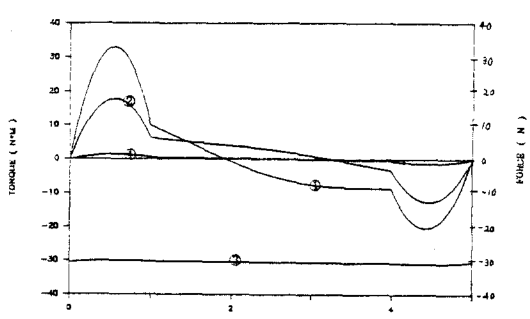

Fig.5 Torque Trajectory

Table.2 Estimation Results

Integration interval : (t_1, t_2) sec

1. (0, 0.5), (0.5, 1.0), (1.0, 1.5)
2. (1.0, 1.5), (1.5, 2.0), (2.5, 3.0)
3. (2.5, 3.0), (3.0, 3.5), (3.5, 4.0)

Sampling time = 2 (msec)

$\hat{X}$	Input data	First Algorithm		
		1	2	3
$\hat{X}_{11}$	17.5500	17.5500	17.5499	17.5492
$\hat{X}_{21}$	7.5500	7.5500	7.5500	7.5497
$\hat{X}_{22}$	2.3430	2.3430	2.3430	2.3430
$\hat{X}_{23}$	0.1760	0.1760	0.1760	0.1758
$\hat{X}_{31}$	3.1100	3.1100	3.1100	3.1100
$\hat{X}_{41}$	0.3500	0.3500	0.3500	0.3500

$\hat{X}$	Input data	Second Algorithm		
		1	2	3
$\hat{X}_{11}$	17.5500	17.5500	17.5502	17.5501
$\hat{X}_{21}$	7.5500	7.5500	7.5500	7.5498
$\hat{X}_{22}$	2.3430	2.3430	2.3430	2.3429
$\hat{X}_{23}$	0.1760	0.1760	0.1760	0.1763
$\hat{X}_{31}$	3.1100	3.1100	3.1100	3.1100
$\hat{X}_{41}$	0.3500	0.3500	0.3500	0.3500

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