

## A study on the slitted parallel-plate waveguide

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한 면에 슬-릿 (slit) 이 있는 평행 평판 도파관에 관한 연구

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### ABSTRACT

A parallel-plate waveguide with a slit in its upper plate is analysed. The magnetic current induced in the slit is obtained by making use of the conventional moment method.

From knowledge of the magnetic current, quantities of interest such as reflection coefficient and transmission coefficient are computed.

### 1. INTRODUCTION

The analysis of the slit in the parallel-plate waveguide was undertaken many years ago by R.F. Millar<sup>(1)</sup> but the results of this early work are valid only for wide slit.

Recently, some authors<sup>(2,3)</sup> considered the characteristics of a slit in the parallel-plate waveguide filled with a truncated dielectric using moment method and some authors<sup>(3),(4)</sup> analysed the narrowly slitted parallel-plate waveguide filled with a homogeneous media.

Here, an efficient computation technique in solving the transverse slit in the wall of a parallel-plate waveguide is presented.

### II. FORMULATION OF THE PROBLEM

When a TEM wave whose electric field amplitude is assumed to be unity is incident upon a slit as shown in Fig.1, electromagnetic fields scattered from the slit may be calculated from an equivalent magnetic surface current in the slit. The time dependence factor  $e^{j\omega t}$  is suppressed throughout and  $k \ll \pi$  such that only the TEM mode can propagate along the waveguide.

Choosing the appropriate Neumann Green function in each region I, II in Fig.1 and imposing the continuity of an electromagnetic field in the slit, one can obtain.<sup>(2)</sup>

$$\frac{k}{2\eta} \int_{-a}^a My(z') H_0^{(2)}(k|z-z'|) dz' = \int_{-a}^a My(z') G(z, z') dz' + \frac{1}{\eta} e^{-j\omega z} \begin{matrix} -a \leq z, z' \leq a \\ x, x' = 0 \end{matrix} \quad (1)$$

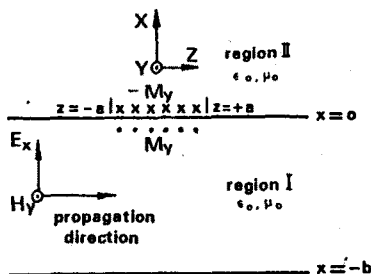


Fig.1 Equivalent magnetic currents in the slit

Where  $k = w \sqrt{\mu_0 \epsilon_0}$ ,  $\eta = \sqrt{\mu_0 / \epsilon_0}$ ,  $w$ ,  $\mu_0$  and  $\epsilon_0$  denote angular frequency, permeability and permittivity respectively and  $H_0^{(2)}$  means the Hankel function of the second kind of index zero, and  $G(z, z')$  means the Green function<sup>(2)</sup> in region I.

The unknown  $My(z')$  is approximated by pulses and the resulting approximate equation is subjected to collocation in conventional moment method.

By partitioning the interval  $(-a, +a)$  into  $N$  segments of length  $\Delta = \frac{2a}{N}$  and by selecting the match-point locations  $Z_m$  at pulse centers according to  $Z_m = -a + (m - \frac{1}{2}) \Delta$ , one may establish the following approximation of the integral equation,

$$\sum_{n=1}^N M_{z,n} [Y_{mn}^1 + Y_{mn}^2] = H_{z,inc}(Z_m) = \frac{1}{\eta} e^{-j k z_m} \quad (2)$$

$m = 1, 2, 3 \dots N$   
 $n = 1, 2, 3 \dots N$   
 $Z_m$ ; observation point.  
 $Z_n$ ; source point.

in which  $My,n$  is the unknown coefficient of the  $n$ th pulse located at  $Z_n$ .

The matrix elements in (2) are defined as

$$Y_{m,n}^I = - \int_{z_n - \Delta/2}^{z_n + \Delta/2} G(z', z_m) dz' \quad (3)$$

$$Y_{m,n}^{II} = \frac{k}{2\eta} \int_{z_n - \Delta/2}^{z_n + \Delta/2} H_0^{(2)}(k|z' - z_m|) dz' \quad (4)$$

Green function  $G(z, z')$  in the slit in equation (3) can be replaced by the small argument approximation<sup>[1]</sup> near the singular point  $z = z'$  as follows.

$$G(z, z') = - \frac{e^{-jk|z-z'|}}{2\eta b} + \frac{jk}{\eta\pi} \left\{ \ln \left( \frac{\pi|z-z'|}{b} \right) - \sum_0^{\infty} \left( \frac{kb}{\pi} \right)^2 \right\} \quad (5)$$

Therefore, in case of  $m = n$ , equation(3) becomes after some algebraic manipulations,

$$Y_{m,n}^I = \frac{1 - e^{-jkD/2}}{j\eta kb} - \frac{jkD}{\eta\pi} \left\{ \ln \left( \frac{D\pi}{2eb} \right) - \sum_0^{\infty} \left( \frac{kb}{\pi} \right)^2 \right\} \quad (6)$$

where  $\sum_0^{\infty} (x) = \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n^2 - x^2}} - \frac{1}{n} \right) : |x| < 1$

For  $m \neq n$ , equation(3) is represented by<sup>[5]</sup>

$$Y_{m,n}^I = \frac{1}{\eta kb} \sin(kD/2) e^{-jk|m-n|D} - j \frac{kb}{\pi^2 \eta} \left\{ f(\pi[|m-n| + 1/2]D/b) - f(\pi[|m-n| - 1/2]D/b) + \sum_{n=1}^{\infty} \left\{ \left( \frac{1}{n^2 - \left( \frac{kb}{\pi} \right)^2} e^{-\pi \sqrt{n^2 - \left( \frac{kb}{\pi} \right)^2} [ |m-n| + 1/2 ] D/b} - \frac{1}{n^2 - 1/4} e^{-n\pi [ |m-n| + 1/2 ] D/b} ) - \left( \frac{1}{n^2 - \left( \frac{kb}{\pi} \right)^2} e^{-\pi \sqrt{n^2 - \left( \frac{kb}{\pi} \right)^2} [ |m-n| - 1/2 ] D/b} - \frac{1}{n^2 - 1/4} e^{-n\pi [ |m-n| - 1/2 ] D/b} ) \right\} \right\} \quad (7)$$

here  $f(x) = 2[1 - \sinh(x/2) \ln(\coth(x/4))]$ ,  $x > 0$

Evaluation of integral of equation(4) for both  $m = n$  case and  $m \neq n$  case is performed by standard technique and omitted here.

Solving the linear simultaneous equation whose coefficient elements  $Y_{m,n}^I, Y_{m,n}^{II}$  are given above, one

obtains the unknown coefficients  $My,n$ .

From knowledge of  $My,n$ , the magnetic field reflection coefficient  $\Gamma_H$  and transmission coefficient  $T_H$  are readily found to be

$$\Gamma_H = - \frac{\sin(kD/2)}{kb} \sum_{n=1}^N My,n e^{-jkz_n} \quad (8)$$

$$T_H = 1 - \frac{\sin(kD/2)}{kb} \sum_{n=1}^N My,n e^{-jkz_n} \quad (9)$$

### III. RESULTS

From the equivalent magnetic current, the magnetic field reflection coefficient  $\Gamma_H$  and transmission coefficient  $T_H$  can be calculated. Values of  $|\Gamma_H|$  and  $|T_H|$  obtained here are illustrated in Fig. 2 and Fig.3 respectively and compared with those of Millar's and Simmons' for the slit range;  $2ka \geq 2$

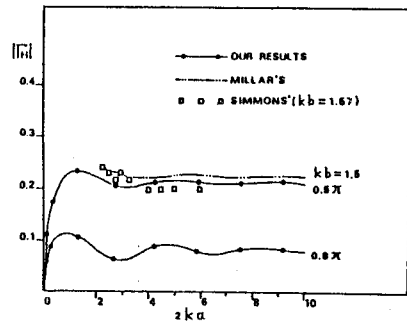


Fig.2 The magnetic field reflection coefficient  $|\Gamma_H|$  with slit width

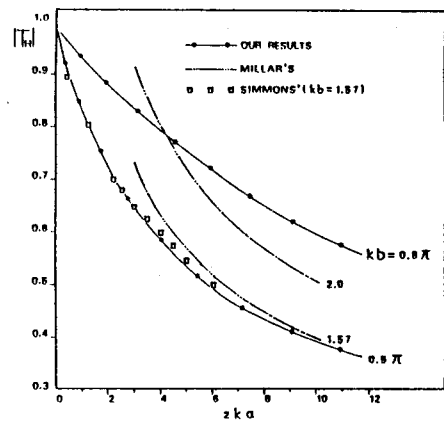


Fig.3 The transmission coefficient  $|T_H|$  with slit width

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