Performance of Coherent Frequency-Hopped Spread-Spectrum Multiple-Access Communication Systems

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Abstract

The error probability for coherent frequency-hopped spread-spectrum multiple-access communications is computed. We employ PSK as a modulation scheme. The channel capacity is performance measure and the minimum bit signal-to-noise ratio for reliable communications is obtained.

1. Introduction

In some communication systems (e.g. satellite communication or mobile communication), many users share the common channel to communicate each other. In this situation it is hard to avoid the mutual interference with unwanted signals at a specified receiver. The frequency-hopped spread-spectrum system is well known to be one of the available methods to overcome the interference among the various user's signals [3,5].

The frequency-hopped communication system uses a bandwidth much greater than actually required for communication [3]. In frequency-hopped spread-spectrum multiple-access (FH/SSMA) communication systems there are two different phenomena which contribute to errors. First, in the absence of noise, errors may occur when a signal is hopped to a frequency slot that is occupied by another signal. Whenever two different signals simultaneously occupy one frequency slot, a hit occurs. Second, in the absence of hits, errors may occur due to the unavoidable background noise.

The bit error probability P_e in FH/SSMA communication systems can be written as

$$P_e = P_o (1-q)^{K-1} + P_1 [1-(1-q)^{K-1}] \qquad (1.1)$$

where K is the number of users and P_o is the conditional probability of error for the observation bit given that there are no hits and P_1 is the conditional probability of error for the observation bit given that there is at least one hit, and q is the probability of hit from one other user. The probability q is depending upon the hopping pattern and several hopping patterns have been introduced in [2]. In coherent system with binary signalling scheme [4], the conditional error probability given no hit is given by

$$P_o = Q\left(\sqrt{2E_c / N_o}\right) \qquad (1.2)$$

where $$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

E_c is signal energy and N_o is one-sided noise spectral density. In (1.1) the only problem to be solved is P_1. In fact, it is very complicated to calculate this conditional probability. In this paper we present the exact conditional density function of the output of correlation receiver given hit by using the characteristic function method for coherent communication systems, and we found that P_1 is close to one half. We consider the channel capacity as a performance measure.

In section 2 we state the channel models and assumptions that we will make for this paper. In section 3 we analyze the performance of the system by using the characteristic function method. Finally in section 4 we draw a conclusion.

2. Channel Models

Before we analyze the performance of the system, we state the assumptions that will be made:
(a) The channel has an additive white Gaussian noise (AWGN) with mean 0 and two-sided power spectral density $N_o/2$.
(b) Every user employs the same digital modulation scheme and every signal is independent of all other signals.
(c) Input signal duration T is equal to frequency hopping signal duration T_h , that is, we assume one bit per hop.
(d) Without loss of generality we consider the first input symbol during the observation time $0 \le t \le T$ for analysis.

(e) Input data symbols are equally probable, i.e.

$$Pr[b^{(i)}=+1] = Pr[b^{(i)}=-1] = 1/2$$

(f) The communication network is assumed to be asynchronous in time.
(g) Coherent demodulation with perfect phase reference is employed.

The transmitter for the frequency hopped spread spectrum signal is shown in Fig. 1. There are K transmitters in the spread spectrum multiple access system and we consider the i-th transmitter and the i-th receiver pair. The i-th data signal $b_i(t)$ is a sequence of duration T. The amplitude of the i-th signal is denoted by $b^{(i)}$ (i.e. $b_i(t)=b^{(i)}$ for $0 \le t \le T$) and $b^{(i)}$ is either +1 or -1. The data signal $b_i(t)$ is the input to PSK modulator, and the output is given by

$$C_i(t)=\sqrt{2E_c/T}\; b_i(t) \cos(2\pi f_c t + \psi_i) \qquad (2.1)$$

where f_c is carrier frequency, ψ_i is phase for the i-th transmitter, and E_c is channel symbol energy.

In Fig. 1, after modulation, the PSK signal is frequency-hopped according to the i-th hopping pattern $f_i(t)$ which is derived from a sequence $\{f_m^{(i)}\}$ = ... , $f_{-1}^{(i)}$, $f_0^{(i)}$, $f_1^{(i)}$, ... according to

$$f_i(t)=f_m^{(i)}, \quad mT_h \le t \le (m+1)T_h \qquad (2.2)$$

T_h is the time between hops. If K hopping patterns are mutually independent and identically distributed, and hopping frequencies are independent every T_h , then the probability of hit, denoted by q, is given by

$$\begin{aligned} q &= 1/v\ (\,2 - 1/v\,) \qquad (2.3) \\ &= 2/v \qquad \text{if } v \text{ is large} \end{aligned}$$

where v is the number of hopping frequencies.

The signal $S_i(t)$, ignoring the unwanted frequency components, is

$$S_i(t)=\sqrt{\frac{2E_c}{T}}\; b_i(t) \cos\left[2\pi(f_c+f_i(t))t+\psi_i+\alpha_i\right] \qquad (2.4)$$

where α_i is the phase introduced by the frequency hopper.

In an asynchronous multiple access system, there exists an arbitrary time delay τ_i for the i-th receiver ($1 \le i \le K$). The received signals are then $S_i(t-\tau_i)$, $1 \le i \le K$. For random hopping patterns we consider time delays modulo T_h , and assume that time delay τ_i is uniformly distributed between 0 and T_h . Similarly, consider phase angle modulo 2π and assume that it is uniformly distributed between 0 and 2π .

We consider the i-th receiver and select the time reference such that $\tau_i=0$. Then τ_k's, $1 \le k \le K$, $k \ne i$, are time delays relative to this time reference.

The i-th receiver model is shown in Fig. 2. The received signal r(t) is given by

$$r(t) = \sum_{k=1}^{K} S_k(t-\tau_k) + n(t) \qquad (2.5)$$

where n(t) is AWGN.

A bandpass filter is designed to remove an unwanted frequency terms such as the double frequency components of the i-th signal, the sum and difference frequency components due to the other K-1 signals and the noise that is outside the frequency band occupied by the i-th signal.

In Fig. 2 β_i is the phase of the dehopping signal and $\hat{\psi}_i$ is the phase of the signal for demodulation. We assume that the time delay, phase angle and data signal are mutually independent. Since it is a coherent system we may assume that $\alpha_i=\beta_i$, $\psi_i=\hat{\psi}_i$. During the observation time interval [0,T] the output of the correlation receiver, r, is given by

$$\begin{aligned} r = b^{(i)} &+ \sum_{\substack{k=1\\k\ne i}}^{K} \frac{1}{T}\int_0^T b_k(t-\tau_k)\cos\left[2\pi(f_k(t-\tau_k)-f_i(t))t\right. \\ &\left. -2\pi(f_c+f_k(t-\tau_k))\tau_k+\phi_k-\beta_i-\psi_i\right]dt \qquad (2.6) \\ &+\sqrt{\frac{2}{E_c T}}\int_0^T n(t)\cos\left[2\pi(f_i(t)-f_c)t+\beta_i-\psi_i\right]dt \end{aligned}$$

where $\phi_k = \psi_k + \alpha_k$

Let X be a channel input (i.e. +1 or -1) and Y be a channel output. The channel output Y depends on the received signal r in Fig. 2, that is, if $r>0$ then Y=+1 and if $r<0$ then Y=-1. Since the channel considered is binary symmetric channel (BSC), the channel capacity is given by

$$C=1+p\,\log_2 p+(1-p)\,\log_2(1-p) \qquad (2.7)$$

where p is the transition probability of BSC. The channel capacity is depending upon K, v, and signal-to-noise ratio $\lambda=E_c/N_0$.

3. Characteristic Function Method

In this section we compute the probability density function given hit by using the characteristic function method [1].

Let I_i be the multiple access interference term, that is, second term in (2.6).

Let Φ_1 denote the characteristic function under H for the random variable I_i , that is,

$$\Phi_1(u)=E\left\{e^{juI_i}\,\middle|\,H\right\} \qquad (3.1)$$

where E(•) is an expected value, u is a real variable, j is the square root of -1 and H is the event that there is a hit.

Let Φ_2 and Φ be the characteristic functions under H for w_i and w_i+I_i, respectively, where w is the Gaussian random variable with zero mean and variance $1/2\lambda$. Because of the symmetry of the distributions of w under H, and I_i under H (and hence of w_i+I_i under H), the characteristic functions Φ_1, Φ_2 and Φ are all real-valued functions. The symmetry of the distributions also implies these characteristic functions are even functions.

Let $f_i(t)$ be $f_m^{(i)}$ during the observation time for i-th receiver and let the hopping frequency of k-th transmitter, $f_k(t-\tau_k)$, be as follows;

$$f_k(t-\tau_k) = \nu_a^{(k)} \quad \text{in} \quad [0, \tau_k]$$
$$f_k(t-\tau_k) = \nu_b^{(k)} \quad \text{in} \quad [\tau_k, T]$$

where $\nu_a^{(k)}$, $\nu_b^{(k)}$ are hopping frequencies and at least one of these frequencies is equal to $f_m^{(i)}$ when there is a hit.

Since w_i and I_i under H are independent, $\Phi(u) = \Phi_1(u)\,\Phi_2(u)$, and

$$\Phi_2(u) = E\left[e^{juw_i}\,|H\right] = e^{-u^2\sigma_0^2/2} \qquad (3.2)$$

where $\sigma_0^2 = 1/2\lambda$, and Φ_1 is given by

$$\Phi_1(u) = E\left[e^{ju\frac{1}{T}\sum_{k=1, k\neq i}^{K} I_{i,k}}\,|H\right] = E\left[\prod_{k=1, k\neq i}^{K} e^{ju\frac{1}{T}I_{i,k}}\,|H\right] \quad (3.3)$$

where

$$I_{i,k} = b_1^{(k)}\,\delta(\nu_a^{(k)}, f_m^{(i)})\cos\eta_{ik}^{(a)}\,\tau_k + b_0^{(k)}\,\delta(\nu_b^{(k)}, f_m^{(i)})\cos\eta_{ik}^{(b)}(T-\tau_k)$$

$$\delta(u,v) = \begin{cases} 1 & u=v \\ 0 & u\neq v \end{cases}$$

and $\eta_{ik}^{(a)}$, $\eta_{ik}^{(b)}$ are new phase terms in the interval $[0,\tau_k]$ and $[\tau_k,T]$, respectively. The simplified form of (3.3) after complicated calculation becomes

$$\Phi_1(u) = \frac{1}{1-(1-q)^{K-1}}\left[M^{K-1} - \left(\frac{v-1}{v}\right)^{2(K-1)}\right] \quad (3.4)$$

where

$$M = \left(\frac{v-1}{v}\right)^2 + 2\hat{A}\,\frac{v-1}{v^2} + B^2\,\frac{1}{v^2}$$

$$\hat{A} = \frac{2}{\pi}\frac{1}{u}\int_0^{\pi/2}\frac{\sin[u\cos\psi]}{\cos\psi}\,d\psi$$

$$\hat{B} = \frac{1}{2}J_0(u) + \frac{1}{\pi}\frac{1}{u}\int_0^{\pi/2}\frac{\sin[u\cos\psi]}{\cos\psi}\,d\psi$$

$$J_0(u) = \frac{1}{\pi}\int_0^{\pi}\cos(u\cos\theta)\,d\theta$$

The characteristic function for w_i+I_i under H is then given by

$$\Phi(u) = \Phi_1(u)\cdot\Phi_2(u) = \Phi_1(u)\cdot e^{-u^2\sigma_0^2/2} \qquad (3.5)$$

By using (3.5) the density function of r given H and $b^{(i)}$, $p(r|H,b^{(i)})$, is given by

$$p(r|H,b^{(i)}) = \frac{1}{\pi}\int_0^{\infty}\cos[u(r-b^{(i)})]\,\Phi(u)\,du \qquad (3.6)$$

The transition probability conditioned on H, p(H), is then given by

$$P(H) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty}\frac{1}{u}\sin(u)\,\Phi(u)\,du \qquad (3.7)$$

The transition probability (i.e. error probability), p, is given by

$$p = (1-q)^{K-1}\,Q\!\left(\frac{1}{\sigma_0}\right) + [1-(1-q)^{K-1}]\,p(H) \quad (3.8)$$

Table 1. Error probability given hit, p(H), when v=100.

K	λ(dB)	p(H)
5	-10	0.487
	0	0.485
	10	0.484
10	-10	0.488
	0	0.485
	10	0.485
15	-10	0.488
	0	0.486
	10	0.486
40	-10	0.491
	0	0.489
	10	0.489

The above Table 1 shows the conditional error probability p(H) when the number of users, K, is 5, 10, 15, 40 , the number of hopping frequencies, v, is 100 and the symbol signal-to-noise ratio, λ, is -10, 0, 10 dB.

Shannon's channel coding theorem says that we can use codes of rates R to communicate reliably over channel provided

$$R < C(\lambda) = C(E_c/N_0) \qquad (3.9)$$

Since E_c is the channel symbol energy and R is the code rate, we can express the information bit energy, E_b, as $E_b = E_c/R$. Thus (3.9) can be rewritten as

$$E_b/N_0 > C^{-1}(R)/R \qquad (3.10)$$

The bit signal-to-noise ratio vs. code rate R is shown in Fig. 3. In Fig. 4 we plot the relation of channel capacity and K given λ

4. Conclusions

Using the results shown in Table 1, Fig. 3 and Fig. 4 we can conclude that the error probability given hit, p(H), is close to one half and when the number of users is fixed (e.g. K=15 in Fig. 3) the code rate R can not be greater than 0.5 even though λ increases. It is because p(H) is almost same for large λ . It is valid for all K. And also we see that λ greater than 7 dB does not improve the channel capacity over all K (see Fig. 4). So the threshold level for λ is around 7 dB in this system.

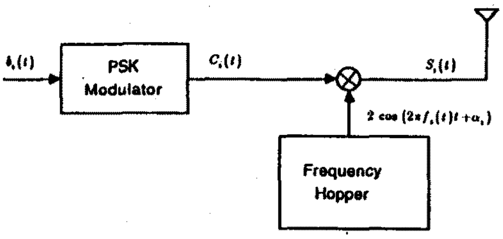

Fig. 1. i-th transmitter model

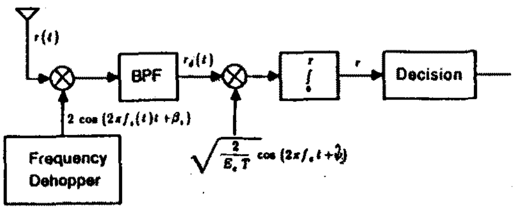

Fig. 2. i-th receiver model

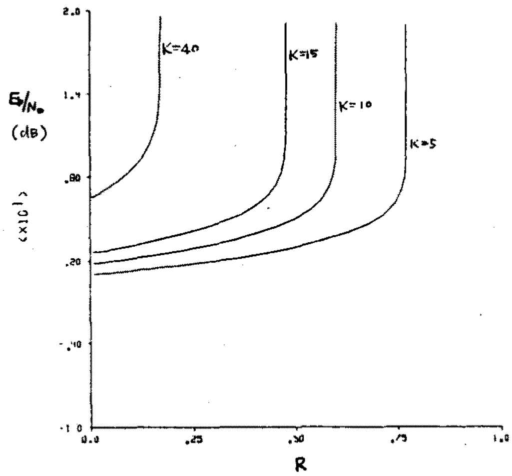

Fig. 3. E_b / N_o needed for reliable communications at code rate R with K=5,10,15,40 and v=100.

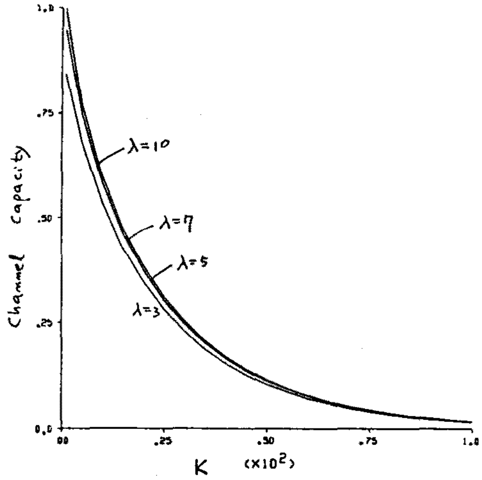

Fig. 4. Channel capacity vs. number of users given λ =3,5,7,10 dB and v=100.

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