A Continuous-time Modified Gain Extended Kalman Filter

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Abstract

A continuous-time modified gain extended Kalman filter (MGEKF) is developed in an effort to extend the discrete-time results of 1) and 2). Used as an observer, it is globally exponentially convergent. For stochastic system, the stability of the MGEKF is proven under certain conditions. The performance of the MGEKF is compared with that of the EKF for a particular nonlinear system where the fininate dimensional optimal filter exists.

1. Continuous-time Deterministic System

A globally convergent observer called the modified gain extended Kalman observer(MGEKO) for a class of continuous-time nonlinear systems is developed in this section.

Consider the deterministic case where the system dynamics and measurements are continuous in time such that the following equations describe the system.

$$\dot{x}_t = f_t(x_t), \quad x_{t_0} = x_0 \tag{1}$$

$$z_t^* = h_t(x_t) \tag{2}$$

where $f(x_t)$ is continuous, bounded and satisfies Lipschitz condition

$$\left\| f_t(x_t) - f_t(y_t) \right\|_d \leq c \left\| x_t - y_t \right\|_d \tag{3}$$

where $\|\cdot\|_d$ means a norm in Euclidean space. The notion of a modifiable nonlinearity is defined as

Definition 1.[1,2] A time-varying function $b_t : R^n \to R^p$ is a modifiable nonlinear system function if there exists a p x n time-varying matrix of functions $c_t : R^q \times R^n \to R^{pxn}$ so that for any x, $\bar{x} \in R^n$ and $t \in R_t$,

$$b_t(x) - b_t(\bar{x}) = c_t(z_t^*, x)(x - \bar{x})$$

where z_t^* satisfies (2).

In order to estimate the states of system of (1) and (2), the following full-order observer, which is in the same form as the extended Kalman observer (EKO), is introduced as

$$\dot{\hat{x}}_t = f_t(\hat{x}_t) + k_t\,(z_t^* - h_t(\hat{x}_t)), \quad x_{t_0} = x_0 \tag{4}$$

Furthermore, if $f_t(\cdot)$ and $h_t(.)$ are modifiable, then the errors in the extimates of the observer of (4) can be written as

$$\dot{e}_t = \dot{x}_t - \dot{\hat{x}}_t = (a_t(z_t^*, \hat{x}_t) - k_t g_t(z_t^*, \hat{x}_t))e_t \tag{5}$$

where

$$f_t(x_t) - f_t(\hat{x}_t) = a_t(z_t^*, \hat{x}_t)(x_t - \hat{x}_t) \tag{6}$$

and

$$h_t(x_t) - h_t(\hat{x}_t) = g_t(z_t^*, \hat{x}_t)(x_t - \hat{x}_t) \tag{7}$$

Since the error equation (5) is written without any approximations and is in the same form as that of a linear system, the gain algorithm is chosen to minimize $tr(\dot{p}_t)$ of the following equation similar to the Kalman filter

$$\dot{p}_t = (a_t - k_t g_t)p_t + p_t(a_t - k_t g_t)^T + Q_t + k_t \gamma_t k_t^T \tag{8}$$

where p_t is pseudo-covariance, and $Q_t \in R^{nxn}$ and $\gamma_t \in R^{qxq}$ are design matrices interpreted as the power spectral density of the process and measurement noises in the stochastic counterpart. If one selects k_t to minimize $tr(\dot{p}_t)$ as in the case of linear observers, k_t would be

$$k_t = p_t g_t(z_t^*, \hat{x}_t)\,\gamma_t^{-1} \tag{9}$$

If (a_t, g_t) is uniformly observable and $(a_t, Q_t^{\frac{1}{2}})$ is uniformly controllable, it can be shown that the error dynamics of (5) are globally convergent to zero by using the Lyapunov function $V_t = e_t^T p_t^{-1} e_t$ in a way that is similar to the approach used by 3).

Remark : The gain of the continuous-time modified gain EKF (MGEKF) in the noisy environment is selected from the algorithm of the continuous-time EKF in order to reduce the biases in the estimates. In this situation k_t satisfies

$$k_t = p_t h_{x_t}(\hat{x}_t)^T \;\gamma_t^{-1} \tag{10}$$

Note that if the above choice is made, the optimality of the observer is sacrificed. However, asymptotic stability of the observer is preserved. This fact will be seen in section 2. Equation (8) can be written with k_t in (10) as

$$\dot{p}_t = a_t p_t + p_t a_t^T + Q_t - p_t h_{x_t}(\hat{x}_t)^T \gamma_t^{-1} g_t p_t - p_t g_t^T \gamma_t^{-1} h_{x_t}(\hat{x}_t) p_t + p_t h_{x_t}(\hat{x}_t)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t) p_t \tag{11}$$

where initial p_0 is a symmetric positive definite matrix. Equation (11) is an unusual Riccati differential equation, although, as will be shown later in section 2, the solution p_t is a symmetric positive definite matrix at all times.

In summary the algorithm of the continuous-time MGEKO for the system of (1) and (2) is given by the propagation equation for the estimate in (4) where k_t is given in (9) and p_t is propagated by (8).

2. Continuous-time Stochastic Systems

Consider the stochastic case where the continuous-time system is corrupted by noises such that the system is expressed in the following Ito form.

$$dx_t = f_t(x_t)dt + d\beta_t \tag{12}$$

$$dy_t = h_t(x_t)\, dt + d\eta_t \tag{13}$$

where β_t and η_t are independent Brownian motion processes with $E\{d\beta_t\, d\beta_t^T\} = Q_t dt$, and $E\{d\eta_t\, d\eta_t^T\} = \gamma_t\, dt$, and $0 < Q_t, \gamma_t < \infty$ for all $t \in R_+$. It is also assumed that $f_t(\cdot)$ of (12) and $h_t(\cdot)$ of (13) satisfy the Lipschitz condition (3), and the following linear growth condition

$$\| f_t(x_t) \|_d \leq k\,(1 + \| x_t \|_d^2) \tag{14}$$

such that the solution of (12) and (13) exists. Furthermore, x_{t_0} is assumed independent of $d\beta_t$ and $d\eta_t$ for all $t \in R_+$. Formally, (13) can be written as

$$z_t \triangleq \frac{dy_t}{dt} \overset{f}{=} h_t(x_t) + v_t \tag{15}$$

where $\overset{f}{=}$ means " is formally equal to " and v_t is a white noise process with $E(v_t v_\tau^T) = \gamma_t\, \delta(t-\tau)$. Furthermore, $f_t(\cdot)$ and $h_t(\cdot)$ are assumed modifiable in order to apply the MGEKF algorithm. Define,

$$z_t^* \triangleq h_t(x_t) \tag{16}$$

such that z_t in (15) becomes

$$z_t = z_t^* + v_t \tag{17}$$

The structure of the continuous-time MGEKF is the same as that of the continuous-time MGEKO described in section 1. However, z_t^* is replaced by z_t to calculate the gain k_t of the MGEKF. The algorithm of the continuous-time MGEKF is summarized below.

$$\dot{\hat{x}}_t \overset{f}{=} f_t(\hat{x}_t) + k_t(\hat{x}_t)(z_t - h_t(\hat{x}_t)) \tag{18}$$

$$k_t(\hat{x}_t) = p_t h_{x_t}(\hat{x}_t)^T \gamma_t^{-1} \tag{19}$$

$$\dot{p}_t \overset{f}{=} a_t(z_t,\hat{x}_t)p_t + p_t a_t(z_t, \hat{x}_t)^T + Q_t - p_t h_{x_t}(\hat{x}_t)^T \gamma_t^{-1} g_t(z_t,\hat{x}_t)p_t - p_t g_t(z_t, \hat{x}_t)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t)p_t + p_t h_{x_t}(\hat{x}_t)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t)p_t \tag{20}$$

An application of the above algorithm in practical situation is illustrated in section 3. This unusual Riccati differential equation (20) also has a symmetric positive definite solution, which is illustrated in Proposition 1.

Proposition 1 : The Riccati differential equation(20) of the MGEKF has a symmetric positive definite solution. (Proof) Refer 4).

As in the case of discrete-time[1,2), the intermediate MGEKF is introduced. The algorithm satisfies

$$\dot{\hat{x}}_t^* \overset{f}{=} f_t(\hat{x}_t^*) + k_t(\hat{x}_t^*)(z_t - h_t(\hat{x}_t^*)) \tag{21}$$

$$k_t(\hat{x}_t^*) = p_t^* h_{x_t}(\hat{x}_t^*)^T \gamma_t^{-1} \tag{22}$$

$$\dot{p}_t^* \overset{f}{=} a_t(z_t^*, \hat{x}_t^*)p_t^* + p_t^* a_t(z_t^*, \hat{x}_t^*)^T + Q_t - p_t^* h_{x_t}(\hat{x}_t^*)^T \gamma_t^{-1} g_t(z_t^*, \hat{x}_t^*)p_t^* - p_t^* g_t(z_t^*, \hat{x}_t^*)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t^*)\, p_t^* + p_t^* h_{x_t}(\hat{x}_t^*)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t^*)\, p_t^* \tag{23}$$

First, the stability of the intermediate MGEKF is analyzed. The errors in the estimates of the intermediate MGEKF can be written, from (12), (13), and (21) as

$$de_t^* = (a_t(z_t^*, \hat{x}_t^*) - k_t(\hat{x}_t^*)g_t(z_t^*, \hat{x}_t^*))e_t^* dt - k_t d\eta_t + d\beta_t \tag{24}$$

where e_t^* is independent of $d\beta_t$ and $d\eta_t$. Consider the following definition.

Definition 2 (5) and 6)) : A continuous stochastic process is said to be exponentially bounded in mean square with esponent δ, if there exist constants $\delta > 0$, $K_1 \geq 0$, and $K_2 > 0$ such that

$$\left\| x_t \right\|^2 \leq K_1 + K_2 e^{-\delta t} \quad \text{for all } t \in R_+$$

where $\|\cdot\|$ is defined in the probabilistic Hilbert space L_2.

A Lyapunov function for the error e_t^* in (24) of the intermediate MGEKF is defined as

$$V_t(e_t^*) = e_t^{*T} P_t^{*-1} e_t^* \qquad (25)$$

Assumption 1 : P_t^{*-1} is assumed bounded from below by a constant matrix $c \cdot I$ such that

$$\left\| V_t(e_t^*) \right\| = \left\| e_t^{*T} P_t^{*-1} e_t^* \right\| \geq c \left\| e_t^* \right\|^2 \qquad (26)$$

Consider the following Theorem.

Theorem 1

The errors in the estimates of the intermediate MGEKF are exponentially bounded in mean square with exponent δ, under the Assumption 1, and for $Q_t \geq \alpha \cdot I > 0$ for all $t \in R_+$.

(proof) Refer 4)

The Ito stochastic differential equation for the errors of the estimates of the MGEKF, determined from (12), (13), and (18), is

$$\begin{aligned} de_t = {} & (a_t(z_t^*, \hat{x}_t) - k_t(\hat{x}_t) g_t(z_t^*, \hat{x}_t)) e_t dt \\ & - k_t(\hat{x}_t) d\eta_t + d\beta_t \end{aligned} \qquad (27)$$

The only difference between the estimates of the continuous-time MGEKF and those of the intermediate MGEKF resulted from the contribution of (V_τ, $0 \leq \tau \leq t$) to the gain of the estimates. Therefore, k_t, a_t, and g_t are affected by such a difference. Define,

$$\Delta k_t = k_t(\hat{x}_t) - k_t(\hat{x}_t^*),$$

$$\Delta c_t \triangleq \Delta(k_t g_t) = c_t(z_t^*, \hat{x}_t) - c_t(z_t^*, \hat{x}_t^*)$$

$$\Delta a_t = a_t(z_t^*, \hat{x}_t) - a_t(z_t^*, \hat{x}_t^*)$$

Consider the following sufficiency Theorem.

Theorem 2

If Δk_t is bounded and Δa_t and Δc_t belong to the set of nondestabilizing deviations such that

$$\begin{aligned} & -(\Delta a_t - \Delta c_t) P_t^* \\ & + \tfrac{1}{2}\left(Q_t + P_t^* h_{x_t}(\hat{x}_t^*)^T \gamma_t^{-1} h_{x_t}(\hat{x}_t^*) P_t^*\right) > 0 \end{aligned} \qquad (28)$$

for all $t \in R_+$

where P_t^* is quantity from the intermediate MGEKF, then the error in the estimates of the continuous-time MGEKF is exponentially bounded with exponent δ.

(Proof) Refer 4)

3. A Comparison of Continuous-time Filters

For a particular nonlinear dynamic system where the finite dimensional optimal filter of Marcus and Willsky[7)] exists, the performances of the MGEKF and the EKF are compared to the optimal performance. Each performance is evaluated by obtaining the analytical solution of P matrix of each filter in order to avoid expensive Monte Carlo simulation. This example also illustrates the way how the continuous-time MGEKF analyzed in section 2 is formulated in practice. Note that P matrices of the EKF and the MGEKF are not the actual covariances of the estimates, however, the point in this section is that the measure to compare the performances is selected as the closeness of P matrices of the EKF and the MGEKF to the optimal one such that Monte Carlo simulation is avoided. Moreover, as shown in 8), it is hard to distinguish the performance of the EKF from that of the optimal estimates by finite number of runs of Monte Carlo simulation.

Consider the system described by

$$\begin{bmatrix} dx_{1_t} \\ dx_{2_t} \end{bmatrix} = \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} \begin{bmatrix} x_{1_t} \\ x_{2_t} \end{bmatrix} dt + q^{\frac{1}{2}} \begin{bmatrix} d\tilde{w}_{1_t} \\ d\tilde{w}_{1_t} \end{bmatrix} \qquad (29)$$

$$dx_{3_t} = -c\, x_{3_t} dt + x_{1_t} x_{2_t} dt \qquad (30)$$

$$\begin{bmatrix} dy_{1_t} \\ dy_{2_t} \end{bmatrix} = \begin{bmatrix} x_{1_t} \\ x_{2_t} \end{bmatrix} dt + r^{\frac{1}{2}} \begin{bmatrix} dw_{1_t} \\ dw_{2_t} \end{bmatrix} \qquad (31)$$

where $a, b, c > 0$, $\tilde{w}_{1_t}$, $\tilde{w}_{2_t}$, w_{1_t}, and w_{2_t} are independent, zero mean unit variance Brownian motion processes, and $x_1(0)$, $x_2(0)$, and $x_3(0)$ are independent random variables which are also independent of the noise processes. It is shown in 7) that the above system is a member of a certain class of nonlinear systems which have the finite dimensional optimal estimators. Note that x_{1_t} and x_{2_t} are states of a linear Gaussian Markov process such that their optimal estimates are the solution of the Kalman-Bucy filter. The conditional state covariance matrix P given $Y^t = \{ y_s, 0 \leq s \leq t \}$ is partitioned as

$$P = \begin{bmatrix} P_{11} & 0 & P_{13} \\ 0 & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} \tag{32}$$

where P_{11}, P_{22} is the solution of Riccati differential equation of the Kalman-Bucy filter of linear systems of (29) and (31).

The analytical performance of the optimal filter of is obtained by taking the expected value of the conditional state covariance matrix P in (32) over Y^t. Therefore, the unconditional expected value of P denoted $\bar{P}$ satisfies

$$\dot{p}_{11} = -2\, ap_{11} + q - \frac{p_{11}^2}{r},\ p_{11_0} \tag{33}$$

$$\dot{p}_{22} = -\, 2bp_{22} + q - \frac{p_{22}^2}{r},\ p_{22_0} \tag{34}$$

$$\dot{\bar{p}}_{33} = -\, 2c\bar{p}_{33} + 2E\{\widehat{x_1 x_2 x_3} - \hat{x}_3\widehat{x_1 x_2}\} - \frac{1}{r}\left[(\overline{p_{13}^2}) + (\overline{p_{23}^2})\right] \text{with } \bar{p}_{33_0} \tag{35}$$

$$\dot{\bar{p}}_{13} = (-a - c - \tfrac{1}{r}p_{11})\bar{p}_{13} + E\{\hat{x}_2\}p_{11},\ \bar{p}_{13_0} = 0 \tag{36}$$

$$\dot{\bar{p}}_{23} = (-b - c - \tfrac{1}{r}p_{22})\bar{p}_{23} + E\{\hat{x}_1\}p_{22},\ \bar{p}_{23_0} = 0 \tag{37}$$

where P_{11} and P_{22} are unaffected by unconditional expectation, since they are deterministic values. Detailed calculation of $\bar{P}_{33}$, $\bar{P}_{13}$, and $\bar{P}_{23}$ is referred to 9).

If the continuous-time EKF is applied to the system of (29), (30), and (31), the estimates $\hat{x}_1$, $\hat{x}_2$, and covariance P_{11}, P_{22} of the EKF are the same as $\hat{x}_1$, $\hat{x}_2$, P_{11}, and P_{22} of the optimal filter because of linear structure of (29) and (31). The estimates of x_3 of the EKF denoted $\hat{x}_3^e$ satisfies

$$d\hat{x}_3^e = (-c\hat{x}_3^e + \hat{x}_1\hat{x}_2)dt + \tfrac{1}{r}p_{13}^e(dy_1 - \hat{x}_1 dt) + \tfrac{1}{r}p_{23}^e(dy_2 - \hat{x}_2 dt) \tag{38}$$

where the superscript e means the quantity is of the EKF.

p_{33}^e, p_{13}^e, and p_{23}^e of the EKF satisfy

$$\dot{p}_{33}^e = -2cp_{33}^e + 2(\hat{x}_2 p_{13}^e + \hat{x}_1 p_{23}^e) - \tfrac{1}{r}(p_{13}^{e\,2} + p_{23}^{e\,2}) \tag{39}$$

$$\dot{p}_{13}^e = (-a - c - \tfrac{1}{r}p_{11})p_{13}^e + \hat{x}_2 p_{11} \tag{40}$$

$$\dot{p}_{23}^e = (-b - c - \tfrac{1}{r}p_{22})p_{23}^e + \hat{x}_1 p_{22} \tag{41}$$

Therefore, the unconditional expected values of p_{13}^e and p_{23}^e are the same as $\bar{p}_{13}$ of (36), and $\bar{p}_{23}$ of (37) respectively, however, $\bar{p}_{33}^e$, which is the unconditional expected value of (39) over Y^t, is determined from

$$\dot{\bar{p}}_{33}^e = -2c\bar{p}_{33}^e + 2E\{\hat{x}_2 p_{13}^e + \hat{x}_1 p_{23}^e\} - \tfrac{1}{r}(S_1 + S_2) \tag{42}$$

where S_1 and S_2 are defined as $S_1 = \overline{p_{13}^{e\,2}}$ and $S_2 = \overline{p_{23}^{e\,2}}$. Then by using Ito's differential rule, the following equations are obtained.

$$\dot{S}_1 = -2(a + c + \tfrac{1}{r}p_{11})S_1 + 2p_{11}R_1,\ S_{1_0} = 0 \tag{43}$$

$$\dot{S}_2 = -2(b + c + \tfrac{1}{r}p_{22})S_2 + 2p_{22}R_2,\ S_{2_0} = 0 \tag{44}$$

where $R_1 = E\{\hat{x}_2 p_{13}^e\}$ and $R_2 = E\{\hat{x}_1 p_{23}^e\}$ such that

$$\dot{R}_1 = -(a + b + c + \tfrac{1}{r}p_{11})R_1 + E\{\hat{x}_2^2\}p_{11},\ R_{1_0} = 0 \tag{45}$$

$$\dot{R}_2 = -(a + b + c + \tfrac{1}{r}p_{22})R_2 + E\{\hat{x}_1^2\}p_{22},\ R_{2_0} = 0 \tag{46}$$

Therefore, (42) can be written as

$$\dot{\bar{p}}_{33}^e = -2c\bar{p}_{33}^e + 2(R_1 + R_2) - \tfrac{1}{r}(S_1 + S_2)\ . \tag{47}$$

Since the system dynamics of (29) and (30) and the measurement equation (31) are modifiable, the continuous-time MGEKF can be applied. For this system, a_t, from $f(x) - f(\hat{x})$, satisfies

$$a_t(z^*, \hat{x}) = \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ x_2 & \hat{x}_1 & -c \end{bmatrix} \text{or} \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ \hat{x}_2 & x_1 & -c \end{bmatrix} \tag{48}$$

and $a_t(z, \hat{x})$ with $(z_1, z_2) \triangleq (\frac{dy_1}{dt}, \frac{dy_2}{dt})$, i.e.,

$$a_t(z, \hat{x}) = \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ z_2 & \hat{x}_1 & -c \end{bmatrix} \text{or} \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ \hat{x}_2 & z_1 & -c \end{bmatrix} \tag{49}$$

is used to calculate the gain and p matrix. Since a_t in (49) is not unique, the first expression for a_t is used in the analysis, however, the second one would bring the results almost same as the first one.

The estimates $\hat{x}_1$, $\hat{x}_2$, and covariances P_{11}, P_{22} of the MGEKF are the same as those of the optimal filter. The estimates $\hat{x}_3^m$ satisfies

$$d\hat{x}_3^m = (-c\hat{x}_3^m + \hat{x}_1\hat{x}_2)dt + \frac{1}{r}(p_{13}^m(dy_1 - \hat{x}_1 dt)) + \frac{1}{r}(p_{23}^m(dy_2 - \hat{x}_2 dt)) \quad (50)$$

where the superscript m means the quantity is of the MGEKF. From the covariance equation of the continuous-time MGEKF, one can obtain

$$dp_{33}^m = -2cp_{33}^m dt + 2(dy_2 p_{13}^m + \hat{x}_1 p_{23}^m dt) - \frac{1}{r}(p_{13}^{m^2} + p_{23}^{m^2})dt = (-2cp_{33}^m + 2(x_2 p_{13}^m + \hat{x}_1 p_{23}^m) - \frac{1}{r}(p_{13}^{m^2} + p_{23}^{m^2}))dt + 2p_{13}^m r dw_2 \quad (51)$$

$$dp_{13}^m = (-a-c-\frac{1}{r}p_{11})p_{13}^m dt + dy_2 p_{11} = ((-a-c-\frac{1}{r}p_{11})p_{13}^m + x_2 p_{11})dt + p_{11} r dw_2 \quad (52)$$

$$dp_{23}^m = ((-b-c-\frac{1}{r}p_{22})p_{23}^m + \hat{x}_1 p_{22})dt \quad (53)$$

As in the case of the optimal filter and the EKF, the unconditional expected values of p_{33}^m, p_{13}^m, and p_{23}^m are taken. $\bar{p}_{23}^m$ is the same as $\bar{p}_{23}$ of the optimal filter and $\bar{p}_{23}^e$ of the EKF, moreover, $\bar{p}_{13}^m$ is the same as $\bar{p}_{13}$ of the optimal filter and $\bar{p}_{13}^e$ of the EKF, since $E\{x_2\} = E\{E\{x_2 \mid Y^t\}\} = E\{\hat{x}_2\}$, and dw_2 is not correlated with deterministic p_{11}. In order to calculate $\bar{p}_{33}^m$, define, $S_3 = (p_{13}^m)^2$ and $S_4 = (p_{23}^m)^2$, then from Ito's differential rule,

$$\dot{S}_3 = -2(a+c+\frac{p_{11}}{r})S_3 + 2p_{11}R_3 + rp_{11}^2, S_{3_0} = 0 \quad (54)$$

where $R_3 = E\{x_2 p_{13}^m\}$ satisfies

$$\dot{R}_3 = -(a+b+c+\frac{p_{11}}{r})R_3 + E\{x_2^2\}p_{11}, R_{3_0} = 0 \quad (55)$$

Since p_{23}^m of the MGEKF is the same as p_{23}^e of the EKF, $S_4 = S_2$, and $\bar{p}_{33}^m$ satisfies

$$\dot{\bar{p}}_{33}^m = -2c\bar{p}_{33}^m + 2(R_3 + R_2) - \frac{1}{r}(S_3 + S_2) \quad (56)$$

Figures 1 through 2 show the results of numerical integration for $\bar{p}_{33}$ of the optimal filter, $\bar{p}_{33}^e$ for the EKF, and $\bar{p}_{33}^m$ for the MGEKF. Initial error covariance matrix for each filter is chosen to be an identity matrix, a=b=c=.9, and the following data are used in the computation.

Table 1

Figure #	q	r	$E\{\hat{x}_{1_0}^2\}$	$E\{\hat{x}_{2_0}^2\}$
1	2	1	0	0
1	2	1	1	1
2	1	1	1	1
2	1	10	1	1

As shown in the Figures, $\bar{p}_{33}^m$ of the MGEKF is closer to the optimal $\bar{p}_{33}$ than $\bar{p}_{33}^e$ of the EKF is. The results with different parameter values show the same trend. Nonzero values of $E\{\hat{x}_{1_0}^2\}$ affects the transient responses, however, the steady state values of the responses are the same for both zero and nonzero values for $E\{\hat{x}_{1_0}^2\}$ and $E\{\hat{x}_{2_0}^2\}$.

4. Conclusion

A globally convergent observer for a class of continuous-time nonlinear deterministic systems is developed. The theory is extended to the stochastic systems and a sufficiency theorem for the stability of the filter (called the MGEKF) is developed. The performance of the continuous-time MGEKF is compared with that of the EKF for a particular nonlinear system where the finite dimensional optimal filter exists. For this system, the previous results[8] show that it is hard to distinguish the covariance of the EKF with that of the optimal filter by finite number of runs of Monte Carlo simulation. The measure to compare the performances is selected as the closeness of P matrices of the filters to the optimal covariance in order to avoid Monte Carlo simulation. The results in this paper show that P matrix of the MGEKF is always closer to the optimal covariance than that of the EKF is. The comparison of true covariances of the filters in analytical ways remains further study.

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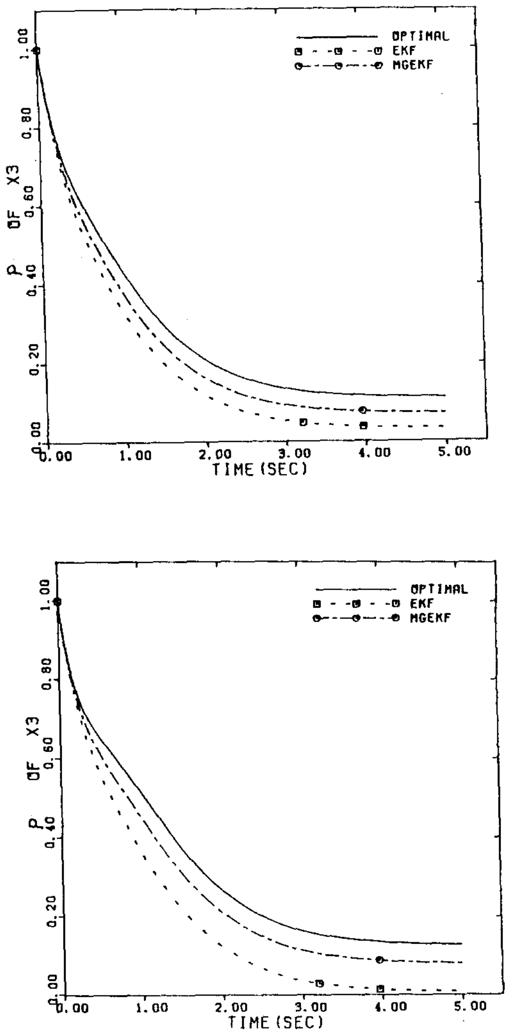

Figure 1. Time histories of $\bar{p}_{33}$, $\bar{p}^{e}_{33}$, and $\bar{p}^{m}_{33}$ when q=1 and r=1 , 10

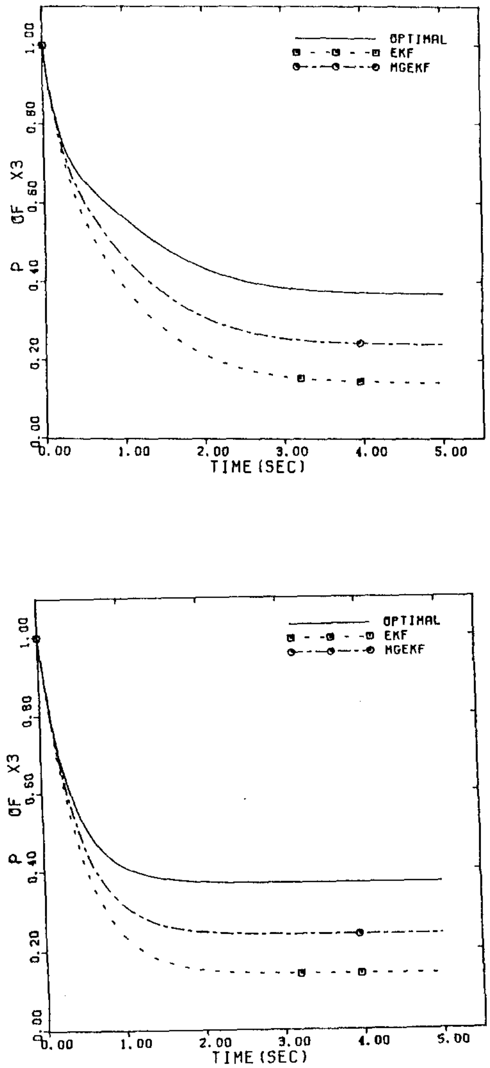

Figure 2. Time histories of $\bar{p}_{33}$, $\bar{p}^{e}_{33}$, and $\bar{p}^{m}_{33}$ when q=2 and r=1 with different initial conditions