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Sensitivity Analysis of the Stability Characteristics

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ABSTRACT

The sensitiveness of the stability characteristics of the aircraft with respect to changes in the stability parameters as predicted by the linear analysis is applied for the two aerodynamic models proposed. The results give the detailed information for an aircraft dynamic behavior especially at complicated flight envelope.

I. INTRODUCTION

Modern high performance aircraft are required to be able to fly and be controlled over the wide variety of flight conditions. To have a tactical advantage over the adversary under these conditions, a well established and detailed analysis of the aerodynamic characteristics of the vehicle are necessary. The obtained static and dynamic stability parameters for an F-5 fighter[1,2,3,4] are applied to general equations of motion for the purpose of describing vehicle motion.

Eigendata and its sensitiveness of the stability characteristics of the F-5 fighter for the two aerodynamic models are investigated. The proposed models are the "combined" model where the oscillatory derivatives ($C_{i_p} + C_{i_{\dot\beta}} \sin\alpha_r$, $C_{i_r} - C_{i_{\dot\beta}} \cos\alpha_r$) are taken to be equal to the pure rotary derivatives (C_{i_p}, C_{i_r}) and the "unsteady" model which includes the pure rotary derivatives and unsteady derivatives ($C_{i_{\dot\beta}}$) individually.

II. EQUATIONS OF MOTION

For the linear model, six general differential equations describing vehicle motion and two kinematic relations representing the vehicle orientation are used. We can list the following assumptions.

(1) Rigid aircraft
(2) Constant mass aircraft
(3) Flat earth
(4) No gyroscopic, jet-damping, buoyancy effects
(5) No structural motion
(6) No control input

The resulting 8-dimensional equations of motion can be written

$$[E]\,\dot{\vec{x}} = [Z]\,\vec{x} \qquad (1)$$

where,

$$\vec{x} = [\ u,\ w,\ q,\ \theta,\ v,\ p,\ r,\ \phi\ ]^T$$

Matrix [E]

m	0	0	0	$-\frac{\partial X}{\partial \dot v}$	0	0	0
0	$m-\frac{\partial Z}{\partial \dot w}$	0	0	$-\frac{\partial Z}{\partial \dot v}$	0	0	0
0	$-\frac{\partial M}{\partial \dot w}$	I_y	0	0	$-I_{xy}-\frac{\partial M}{\partial \dot p}$	$-I_{yz}-\frac{\partial M}{\partial \dot r}$	0
0	0	0	1	0	0	0	0
0	0	0	0	$m-\frac{\partial Y}{\partial \dot v}$	$\frac{\partial Y}{\partial \dot p}$	$\frac{\partial Y}{\partial \dot r}$	0
0	0	$-I_{xy}-\frac{\partial L}{\partial \dot q}$	0	$-\frac{\partial L}{\partial \dot v}$	$I_x-\frac{\partial L}{\partial \dot p}$	$-I_{xz}-\frac{\partial L}{\partial \dot r}$	0
0	0	$-I_{yz}-\frac{\partial N}{\partial \dot q}$	0	$-\frac{\partial N}{\partial \dot v}$	$I_{xz}-\frac{\partial N}{\partial \dot p}$	$-I_z-\frac{\partial N}{\partial \dot r}$	0
0	0	0	0	0	0	0	1

Matrix [Z]

$-mr+\frac{\partial Y}{\partial u}+\frac{\partial T}{\partial u}\sin\sigma\cos\varepsilon$	$mp+\frac{\partial Y}{\partial w}+\frac{\partial T}{\partial w}\sin\sigma\cos\varepsilon$	$\frac{\partial Y}{\partial q}$	$-mg\sin\theta\sin\phi$	$mr+\frac{\partial X}{\partial v}+\frac{\partial T}{\partial v}\cos\sigma\cos\varepsilon$	$\frac{\partial X}{\partial p}$	$mv+\frac{\partial X}{\partial r}$	0
$\frac{\partial L}{\partial u}$	$\frac{\partial L}{\partial w}$	$2I_{xz}q+L_{xy}p+(I_y-I_z)r+\frac{\partial L}{\partial q}$	0	$-mp+\frac{\partial Z}{\partial v}-\frac{\partial T}{\partial v}\sin\varepsilon$	$-mv+\frac{\partial Z}{\partial p}$	$\frac{\partial Z}{\partial r}$	$-mg\cos\theta\sin\phi$
$\frac{\partial N}{\partial u}$	$\frac{\partial N}{\partial w}$	$-2I_{xy}q-I_{xz}r+(I_x-I_y)p+\frac{\partial N}{\partial q}$	0	$\frac{\partial M}{\partial v}$	$-2I_{xz}p-I_{yz}q+(I_z-I_x)r+\frac{\partial M}{\partial p}$	$2I_{xz}r+I_{xy}q+(I_x-I_z)p+\frac{\partial M}{\partial r}$	0
0	0	$\sin\phi\tan\theta$	$\frac{q\sin\phi+r\cos\phi}{\cos^2\theta}$	0	0	$-\sin\phi$	$-q\sin\phi-r\cos\phi$
$\frac{\partial X}{\partial u}+\frac{\partial T}{\partial u}\cos\sigma\cos\varepsilon$	$-mq+\frac{\partial X}{\partial w}+\frac{\partial T}{\partial w}\cos\sigma\cos\varepsilon$	$-mw+\frac{\partial X}{\partial q}$	$-mg\cos\theta$	$\frac{\partial Y}{\partial v}+\frac{\partial T}{\partial v}\sin\sigma\cos\varepsilon$	$mw+\frac{\partial Y}{\partial p}$	$-mu+\frac{\partial Y}{\partial r}$	$mg\cos\theta\cos\phi$
$mq+\frac{\partial Z}{\partial u}-\frac{\partial T}{\partial u}\sin\varepsilon$	$\frac{\partial Z}{\partial w}-\frac{\partial T}{\partial w}\sin\varepsilon$	$mu+\frac{\partial Z}{\partial q}$	$-mg\sin\theta\cos\phi$	$\frac{\partial L}{\partial v}$	$I_{xz}q-I_{xy}r+\frac{\partial L}{\partial p}$	$-2I_{yz}r-I_{xy}p-(I_y-I_z)q+\frac{\partial L}{\partial r}$	0
$\frac{\partial M}{\partial u}$	$\frac{\partial M}{\partial w}$	$-I_{yz}p+I_{xy}r+\frac{\partial M}{\partial q}$	0	$\frac{\partial N}{\partial v}$	$2I_{xy}p+I_{yz}r+(I_x-I_y)q+\frac{\partial N}{\partial p}$	$I_{yz}p-I_{xz}q+\frac{\partial N}{\partial r}$	0
0	0	$\cos\phi$	0	0	1	$\cos\phi\tan\theta$	$\tan\theta(q\cos\phi-r\sin\phi)$

Upon linearization, the system is transformed into conventional aircraft variable such that the state is

$$\vec{x} = \{ V, \alpha, q, \theta, \beta, p, r, \phi \}^T \qquad (2)$$

Then, a quasi-nondimensional system is defined such that the state variables have following form

$$\vec{x} = \{ \hat{V}, \alpha, \hat{q}, \theta, \beta, \hat{p}, \hat{r}, \phi \}^T \qquad (3)$$

By using the nondimensional stability data obtained at a particular reference equilibrium point, conventional eigenvalue and eigenvector analysis to the given matrix system yields the system characteristics.

The following equation is the derivative of the eigenvalue with respect to the parameter which appears in the element of E and z matrices.

$$\frac{\partial \lambda_i}{\partial \alpha} = \vec{y}_i^T \left(- \lambda_i \frac{\partial [E]}{\partial \alpha} + \frac{\partial [Z]}{\partial \alpha} \right) \vec{x}_i \qquad (4)$$

$$i = 1, 2, 3 \, ... \, n$$

Detailed methods for calculating the derivatives of the eigenvalues are presented in References 5,6,7 and 8.

III. METHOD OF ANALYSIS

The need for reliable theoretical prediction of vehicle motion especially at out of flight requires better understanding of the complex flow phenomena and its relationship to the characteristics of vehicle motion.

The purpose is to look at an analytical method for predicting vehicle motions and the effects on these predicted motions due to a change in the aerodynamic parameters.

It has been shown that application of a linear model to obtain the stability characteristics of an aircraft is useful in describing the complex motions corresponding to out-of-control flight. This same linear model can also prove useful for demonstrating variation on stability(9).

Some success has been achieved with the linear formulation in that lineary predicted stability derivatives lead to valuable insight of the a aircraft motions throughtout the aerodynamic angles α and β respectively(10).

The sensitivity of the eigenvalues with respect to the elements (stability derivatives) of the system matrix, make it possible to calculate the importance of various parameters on system characteristics. Furthermore these derivatives can be used to make a preliminary estimation of the changes in the characteristics of vehicle motion due to change in the parameters of interest.

IV. RESULT AND DISCUSSIONS

For a straight and level flight reference condition with equilibrium angles of attack from 30^o through 43^o (unsteady model) or 23^o throght 43^o (combined model), the Dutch roll mode is unstable(Figure 1). The on-set of directional divergence near 31^o and the unstable rolling convergence mode above 35^o in unsteady model correspond well with flight test results(11).

Eigendata sensitivity comparisons for unsteady and combined model are made throughout the angle of attack range from 0^o to 45^o.

At lower angles of attack the results of two models for Dutch roll mode are in good agreement for 12 different eigendata, $\partial \lambda / \partial C_{i_j}$ ($C_i = C_y, C_l, C_n$, $j = \beta, p, \dot{\beta}, \dot{p}$).

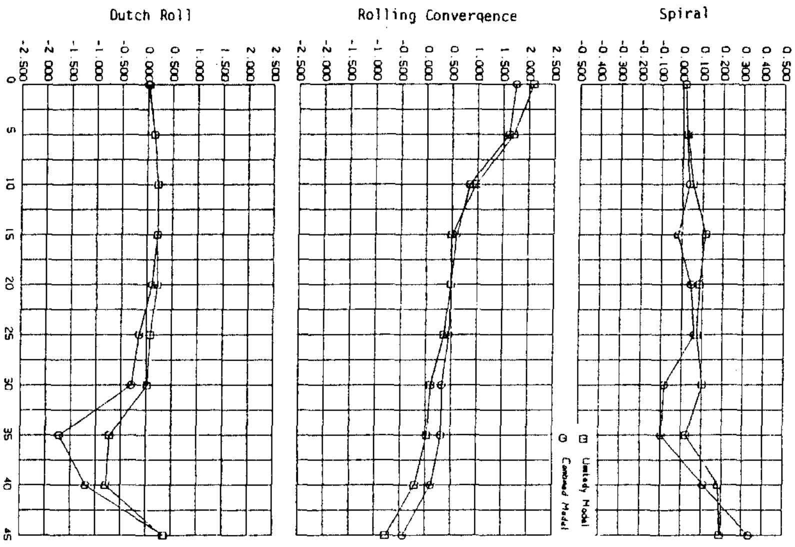

Figure 1. Comparison of Dynamic Stability Characteristics. ($1/t_{1/2}$)

Table 1. Eigendata Sensitivity Comparisons for Different Models (Dutch Roll).

para. / model / α (deg)	$\frac{\delta\lambda}{\delta C_{l_\beta}}$		$\frac{\delta\lambda}{\delta C_{n_\beta}}$		$\frac{\delta\lambda}{\delta C_{l_p}}$		$\frac{\delta\lambda}{\delta C_{n_p}}$		$\frac{\delta\lambda}{\delta C_{l_{\dot p}}}$	
	unsteady	combined	unsteady	combined	unsteady	combined	unsteady	combined	unsteady	combined
25	.761 -15.460 i	1.503 -13.883 i	.017 +1.885 i	-.535 +1.565 i	2.189 + .465 i	1.977 + .403 i	-.269 -.041 i	-.218 -.098 i	-.30 +.122i	-.017 +.114i
30	.071 - 8.540 i	.049 - 8.451 i	.203 + .845 i	-.024 + .816 i	1.772 + .135 i	1.768 + .014 i	-.178 +.030 i	-.171 -.005 i	-.015 +.189i	-.010 +.191i
35	.150 - 7.149 i	- .127 -. 7.257i	.252 +.611 i	-.116 + .536 i	1.689 - .075 i	1.690 - .390 i	-.140 +.069 i	-.131 -.0005 i	.038 +.238i	.121 +.223i
40	.234 -10.872 i	- .396 -10.973 i	.003 +.004 i	-.0001 + .0045 i	1.537 - .151 i	1.549 - .375 i	-.105 +.032 i	-.101 +.001 i	.043 +.156i	.081 +.150i
45	-.574 -13.245 i	- .144 -13.398 i	-.109 +.793 i	-.029 + .769 i	1.335 + .270 i	1.315 + .398 i	-.076 -.030 i	-.074 -.027 i	-.034 +.123i	-.048 +.119i

Table 2. Eigendata Sensitivity Comparisons for Different Models(Rolling Convergence)

Para. / Model / α (deg)	$\frac{\delta\lambda}{\delta C_{l_\beta}}$		$\frac{\delta\lambda}{\delta C_{n_\beta}}$		$\frac{\delta\lambda}{\delta C_{l_{\dot\beta}}}$		$\frac{\delta\lambda}{\delta C_{n_{\dot\beta}}}$	
	unsteady	**combined**	**ussteady**	**combined**	unsteady	combined	unsteady	combined
15	.096	-.141	.033	-.0472	.001	-.002	-.0004	.0006
20	.075	.344	.045	.205	-.0008	-.004	-.0005	-.002
25	.085	-.186	.213	.532	-.0007	.002	.002	-.006

As shown in Table 1, the $\partial\lambda/\partial C_{l_\beta}$, $\partial\lambda/\partial C_{n_\beta}$ and $\partial\lambda/\partial C_{l_{\dot{p}}}$ data between two models are significantly different at higher angles of attack. For rolling convergence and spiral mode sensitivity data(Table 2,3), the difference can be found above 15 degrees angle of attack. The key observation here is that at certain angle of attack ranges the sensitivity of the real part of the eigenvalue to changles in stability derivatives are different in sign. Hence the effect of these parameters on stability is the opposite, depending upon the aerodynamic model used.

V. CONCLUSION

1. Compared to the "combined" aerodynamic model data, the "unsteady" aerodynamic model data yields better agreement with the full-scale flight test results at high angles of attack.

2. Eigendata, along with the sensitivity analysis of the fully coupled linearized equations of motion using the two aerodynamic models indicate that at certain flight conditions the two models give opposite results with regard to stability and with regard to changes in stability due to changes in selected stability parameters.

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Table 3. Eigendata Sensitivity Comparisons for Different Models (Spiral)

Para. / Model / α (deg)	$\frac{\partial\lambda}{\partial C_{l_\beta}}$		$\frac{\partial\lambda}{\partial C_{n_\beta}}$		$\frac{\partial\lambda}{\partial C_{l_p}}$		$\frac{\partial\lambda}{\partial C_{n_p}}$	
	unsteady	combined	unsteady	combined	unsteady	combined	unsteady	combined
15	.412	-.141	.149	-.047	-.510	1.398	-.184	.469
20	.042	.483	.026	.306	.232	-.135	-.147	-.086
25	-.015	-.048	.037	.133	.083	.053	-.211	-.146

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