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ARMA 스펙트럼 추정을 위한 격자구조 확장기구 변수법

Lattice Implementation of Extended Instrumental Variable Method for ARMA Spectral Estimation

Heung Suk Yang 서울대
Chan Soo Chung 숭전대
Hyun Do Nam* 단국대
Chul Heui Lee 서울대

1. Introduction

Parametric modeling techniques are used extensively for high resolution spectral estimation. In these techniques spectral estimation problem is reduced to estimating the coefficients of the time series model.[1]

The least squares method(LSM), one of the most popular methods for parameter estimation, is very simple but gives biased estimates for correlated noise case. To overcome this shortage, the instrumental variable method(IVM) is proposed.[2] But IVM cannot give estimates of moving average(MA) parameters, so various version of approximate maximum likelihood(AML) method such as recursive maximum likelihood(RML) method, RML1, RML2, and extended least squares(ELS) method are used for estimation of noise dynamics.[3]

In this paper recursive extended instrumental variable(EIV) algorithm and covariance lattice implementation of it using embedding approach are presented. This algorithm can estimate autoregressive(AR) and moving average(MA) parts simultaneously using extended regression vector, where the driving inputs, the innovations are replaced with their best estimates obtained by using the current parameter estimates.

Computer simulation is presented to show the efficiency of this algorithm.

2. Spectral Estimation with EIV

The observed data sequence y(t) can be modeled by ARMA process of order (p,q) with p q.

$$y(t)=-\sum_{i=1}^{p}A(i)y(t-i)+v(t) \tag{1}$$

$$v(t)=\sum_{i=1}^{q}C(i)w(t-i)+w(t) \tag{2}$$

where w(t) is awhite process.

The model above can be written in matrix form as

$$Y=XA+V \tag{3}$$

where

$$A=[A(1),\cdots\cdots,A(p)]^T$$

$$Y=[y(p),\cdots\cdots,y(t)]^T$$

$$X=\begin{bmatrix}-y(p-1),\cdots\cdots,-y(0)\\ \vdots \qquad\qquad \vdots\\ -y(t-1),\cdots\cdots,-y(t-p)\end{bmatrix}$$

$$V=[v(p),\cdots\cdots,v(t)]^T$$

The least squares solution of eq.(3) is

$$\hat{A}=(X^TX)^{-1}X^TY-(X^TX)^{-1}X^TV \tag{4}$$

The second term on the right hand side of eq.(4) is a nonzero bias vector when v(t) is a correlated sequence. This is a severe drawback of LSM.

To overcome this shortage, IVM can be applied. Then the solution of eq.(3) is

$$\hat{A}=(Z^TX)^{-1}Z^TY-(Z^TX)^{-1}Z^TV \qquad (5)$$

If we choose the matrix Z so that $E\{Z^TV\}=0$, the bias term of eq.(5) will be vanished automatically. In ARMA case the instruments are usually the delayed outputs.

Though IVM gives consistent estimates for ARMA process, it cannot estimate MA parameters. To estimate AR and MA parameters simultaneously, the extended instrumental variable(EIV) method is proposed.

If we assume, for the moment, that w(t) is measurable, then the model of eqs. (1) and (2) can be written in matrix form as

$$Y=H\theta+W \qquad (6)$$

where $\theta=[A(1)\cdots A(p)\ C(1)\cdots C(q)]^T$

$$H=\begin{bmatrix} -y(p-1)\cdots -y(0) & w(p-1)\cdots w(p-q) \\ \vdots & \vdots \\ -y(t-1)\cdots -y(t-p) & w(t-1)\cdots w(t-q) \end{bmatrix}$$

$$W=[w(p)\cdots\cdots w(t)]^T$$

In fact the sequence w(t) is not known. However the natural way to proceed is to replace w(t) by e(t) which is residual sequence given by

$$e(t)=y(t)-\phi(t)^T\hat{\theta} \qquad (7)$$

where $\phi(t)\stackrel{T}{=}[-y(t-1)\cdots -y(t-p)\, e(t-1)\cdots e(t-q)]$

$$\hat{\theta}=[\hat{A}(1)\cdots\cdots\hat{A}(p)\ \hat{C}(1)\cdots\cdots\hat{C}(q)]^T$$

The solution of eq.(6) in the least squares sense is

$$\hat{\theta}=(Z^TH)^{-1}Z^TH-(Z^TH)^{-1}Z^TE \qquad (8)$$

where $E=[e(p)\cdots\cdots e(t)]^T$

We must choose the matrix Z so that $E\{Z^TE\}=0$, and $E\{Z^TH\}=R$ is a nonsingular matrix. By choosing Z as eq.(9), the conditions above are automatically satisfied from the consistency condition of IVM and the whiteness property of residual process.[1),3)]

$$Z=\begin{bmatrix} -y(0) & 0\cdots\cdots 0 & e(p)\cdots\cdots e(p-q+1) \\ \vdots & \vdots & \vdots \\ -y(p-1)\cdots & -y(0) & e(2p-1)\cdots e(2p-q) \\ \vdots & \vdots & \vdots \\ -y(t-1-p)\cdots & -y(t-2p) & e(t-1)\cdots e(t-q) \end{bmatrix} \qquad (9)$$

As eq.(6) is solved in the least squares sense, so this leads to the following recursive algorithms.

$$\varepsilon(t)=y(t)-\phi(t)^T\hat{\theta}(t-1) \qquad (10)$$

: prediction error

$$P(t)=\frac{P(t-1)(I-Z(t)\phi(t)^TP(t-1))}{\lambda(t)(\lambda(t)+\phi(t)^TP(t-1)Z(t))} \qquad (11)$$

$$K(t)=\frac{P(t)Z(t)}{\lambda(t)+\phi(t)^TP(t)Z(t)} \qquad (12)$$

$$\hat{\theta}(t)=\hat{\theta}(t-1)+K(t)\varepsilon(t) \qquad (13)$$

$$e(t)=y(t)-\phi(t)^T\hat{\theta}(t) \qquad (14)$$

where $\lambda(t)$ is a forgetting factor,

$$Z(t)\stackrel{T}{=}[-y(t-1-p)\cdots -y(t-2p)\, e(t-1)\cdots e(t-q)]$$

3. Lattice Implementation of EIV

The prediction model can be realized in many different ways. Lattice form realization, one of these, has a number of attractive features.[4)] It is computationally efficient, and has an orthogonality property and good numerical features.

Lattice algorithms have different form by choosing the type of windowing the observed data,i.e. autocorrelation form, prewindowed form, postwindowed form, and covariance form.[5)]

The recursive instrumental variable lattice algorithm for covariance case was derived in 6), where the projection operator was used. Here the covariance lattice IV algorithm for ARMA process using embedding approach is presented.

ARMA modeling problem is reduced to solving a two channel AR modeling problem when the embedding approach is applied.[7)]

The bootstrapping technique is very

powerful approach for driving and implementing recursive estimation schemes. Once the estimates of the current pridiction errors are obtained, they can be treated as known data, hence entered into the input of the ladder forms along with other measured data.[7]

Suppose we are given an ARMA model of the following form:

$$y(t)=-\sum_{i=1}^{p} A(i)y(t-i)+\sum_{i=0}^{q} B(i)u(t-i) \quad (15)$$

where p=q.

When the input and the output are augmented, two channel AR process is

$$\begin{bmatrix} e^y(t) \\ e^u(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} A(i) & -B(i) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y(t-i) \\ u(t-i) \end{bmatrix} \quad (16)$$

In cases where the input u(t) are not known, one can approximate them by their best estimates obtained by using the current parameter estimates.

Thus the algorithm is a two step procedure:

1) Using the new data point, old parameter estimates and the previous prediction errors, compute new prediction error $e^y(t)$.

2) Using y(t) and $e^y(t)$ as the input into a recursive AR parameter estimation algorithm, update the lattice parameters.

This procedure is depicted in Table 1. And the whitening filter is in Table 2.

Once the parameters are estimated, then the spectrum $S(\omega)$ is given by

$$S(\omega)=\frac{B(e^{j\omega})\ B(e^{-j\omega})}{A(e^{j\omega})\ A(e^{-j\omega})}\sigma^2$$

where σ^2 is the variance of the input,

$$A(z)=I+A(1)z^{-1}+\cdots\cdots+A(p)z^{-p}$$

$$B(z)=I+B(1)z^{-1}+\cdots\cdots+B(p)z^{-p}$$

Computer simulations are performed to to show the effectiveness of the proposed algorithms compared to ELS and RML.

In this simulation, the narrow band sinusoidal process corrupted by correlated noise is used.

$$y(t)=\sqrt{6.3}\ \sin(.4\pi t)+\sqrt{20}\ \sin(.42\pi t)+v(t)$$

$$v(t)=w(t)-.102w(t-1)+.173w(t-2)$$

where w(t) is white process with unit variance.

The number of data point is taken as T=512 and ARMA(8,8) model is also used.

4. Conclusion

For high spectral estimation, the EIV algorithm is presented. And the covariance lattice implementation of the EIV is also presented.

This algorithm can estimate ARMA parameter using the given data and innovation process.

In compared with ELS, the computational burden is not increasing, but it gives better resolution than RML as well as ELS.

Reference

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3. T. Söderström, L. Ljung and I. Gustavsson,"A Comparative Study of Recursive Identification Methods", Dept. Automat. Contr., Lund Inst., Technology, Lund, Sweden, Rep. 7427, 1974.
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5. B. Friedlander,"Lattice Implementation of Some Recursive Parameter Estimation Algorithms", in Proc. 6th IFAC symp. Ident. Syst. Para. Estimat., pp.481-486, 1982.

6. H.S. Yang, H.D. Nam and J.K. Kim, "Covariance Lattice Instrumental Variable Algorithm For Spectral Estimation", Tr. of KIEE, Vol.35, No.4, 1986.
7. H.S. Yang, H.D. Nam and C.H. Lee, "ARMA Spectral Estimation Using Covariance Lattice Structure", Proc. KIEE Autumn Conference, 1984.

For I = 0, 1, ⋯ , P DO :

$A_{0,I} = B_{0,I} = 1$, $C_{0,I} = D_{0,I} = 0$ $I = 0$

$A_{0,I} = B_{0,I} = C_{0,I} = D_{0,I} = 0$ $I > 0$

For N = 0, 1, ⋯ , P-1 DO :

$C1_{N,I} = C_{N,I} - h^{z'}_{N,T}(1 - f_{N,T+1})^{-1} D_{N,I}$

$D1_{N,I} = D_{N,I} - h^{x}_{N,T}(1 - g_{N,T+1})^{-1} C_{N,I}$

$A1_{N,I} = A_{N,I} - d^{x}_{N,T}(1 - f_{N,T})^{-1} D1_{N,I-1}$

$B1_{N,I} = B_{N,I} - r^{x}_{N,T}(1 - g1_{N,T+1})^{-1} C1_{N,I}$

$A_{N+1,I} = A1_{N,I} - K^{x\bar{z}}_{N,T}(K^{\bar{z}\bar{z}}_{N,T-1})^{-1} B1_{N,I}$

$B_{N+1,I} = B1_{N,I-1} - K^{\bar{z}x}_{N,T}(K1^{xx}_{N,T})^{-1} A1_{N,T}$

$C_{N+1,I} = C1_{N,I} - r^{z'}_{N,T}(K^{\bar{z}\bar{z}}_{N,T-1})^{-1} B_{N,I}$

$D_{N+1,I} = D1_{N,I-1} - d^{z'}_{N,T}(K^{xx}_{N,T})^{-1} A_{N,I}$

Table 2. The whitening filter for the covariance lattice form of EIV

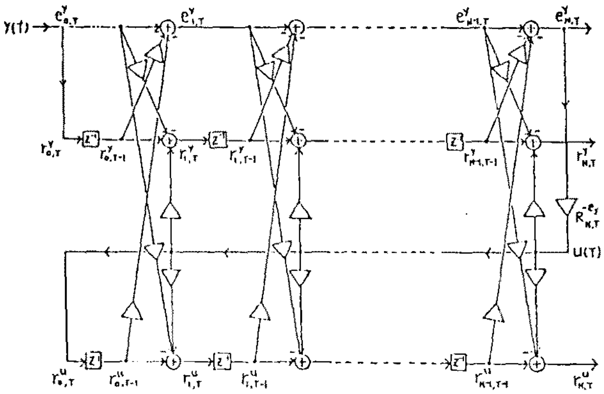

Fig. 1. ARMA lattice form with bootstrap update

Initialization :

$K^{xx}_{0,-1} = K^{\bar{z}\bar{z}}_{0,-1} = \delta I$

Reflection coefficients and backward prediction errors are all initialized to zero.

For T = 0, 1, ⋯ , LT DO :

$e^{x}_{0,T} = r^{x}_{0,T} = \begin{pmatrix} y_T \\ 0 \end{pmatrix}$, $e^{z}_{0,T} = r^{z}_{0,T} = \begin{pmatrix} z_T \\ 0 \end{pmatrix}$

For N = 0, 1, ⋯ , min(P,T)-1 DO :

$e^{x}_{N+1,T} = e1^{x}_{N,T} - K^{x\bar{z}}_{N,T}(K^{\bar{z}\bar{z}}_{N,T-1})^{-1} r^{x}_{N,T-1}$

$r^{x}_{N+1,T} = r^{x}_{N,T-1} - K^{\bar{z}x}_{N,T}(K1^{xx}_{N,T})^{-1} e1^{x}_{N,T}$

$e^{x}_{0,T} = r^{x}_{0,T} = \begin{pmatrix} y_T \\ e^x_{P,T} \end{pmatrix}$, $e^{z}_{0,T} = r^{z}_{0,T} = \begin{pmatrix} z_T \\ e^x_{P,T} \end{pmatrix}$

$f_{0,T} = g_{0,T} = h^{x}_{0,T} = h^{z}_{0,T} = 0$

$K^{xz}_{0,T} = K^{\bar{z}\bar{z}}_{0,T} = K^{xz}_{0,T-1} + \begin{pmatrix} y_T \\ e^x_{P,T} \end{pmatrix} (z_T \; e^x_{P,T})$

For N = 0, 1, ⋯ , min(P,T)-1 DO :

$d^{x}_{N,T} = d^{x}_{N,T-1} + e^{x}_{N,T}(1 - g_{N,T})^{-1} h^{z'}_{N,T-1}$

$d^{z}_{N,T} = d^{z}_{N,T-1} + e^{z}_{N,T}(1 - g_{N,T})^{-1} h^{x'}_{N,T-1}$

$e1^{x}_{N,T} = e^{x}_{N,T} - d^{x}_{N,T}(1 - f_{N,T})^{-1} h^{x}_{N,T-1}$

$e1^{z}_{N,T} = e^{z}_{N,T} - d^{z}_{N,T}(1 - f_{N,T})^{-1} h^{z}_{N,T-1}$

$g1_{N,T} = g_{N,T} + h^{x}_{N,T-1}(1 - f_{N,T})^{-1} h^{x}_{N,T-1}$

$K^{x\bar{z}}_{N,T} = K^{x\bar{z}}_{N,T-1} + e1^{x}_{N,T}(1 - g1_{N,T})^{-1} r^{z}_{N,T-1}$

$K^{\bar{z}x}_{N,T} = K^{\bar{z}x}_{N,T-1} + r^{x}_{N,T-1}(1 - g1_{N,T})^{-1} e1^{z'}_{N,T}$

$g_{N+1,T+1} = g_{N,T} + e^{z'}_{N,T}(K^{xz}_{N,T})^{-1} e^{x}_{N,T-1}$

$h^{x}_{N+1,T} = h^{x}_{N,T-1} - d^{z'}_{N,T}(K^{xz}_{N,T})^{-1} e^{x}_{N,T}$

$h^{z}_{N+1,T} = h^{z}_{N,T-1} - d^{x'}_{N,T}(K^{xz}_{N,T})^{-1} e^{z}_{N,T}$

$f_{N+1,T+1} = f_{N,T} + d^{z'}_{N,T}(K^{xz}_{N,T})^{-1} d^{x}_{N,T}$

$K1^{xz}_{N,T} = K^{xz}_{N,T} - d^{x}_{N,T}(1 - f_{N,T})^{-1} d^{z'}_{N,T}$

$e^{x}_{N+1,T} = e1^{x}_{N,T} - K^{x\bar{z}}_{N,T}(K^{\bar{z}\bar{z}}_{N,T-1})^{-1} r^{x}_{N,T-1}$

$e^{z}_{N+1,T} = e1^{z}_{N,T} - K^{\bar{z}z'}_{N,T}(K^{\bar{z}\bar{z}'}_{N,T-1})^{-1} r^{z}_{N,T-1}$

$r^{x}_{N+1,T} = r^{x}_{N,T-1} - K^{\bar{z}x}_{N,T}(K1^{xz}_{N,T})^{-1} e1^{x}_{N,T}$

$r^{z}_{N+1,T} = r^{z}_{N,T-1} - K^{x\bar{z}'}_{N,T}(K1^{xz'}_{N,T})^{-1} e1^{z}_{N,T}$

$K^{xz}_{N+1,T} = K1^{xz}_{N,T} - K^{x\bar{z}}_{N,T}(K^{\bar{z}\bar{z}}_{N,T-1})^{-1} K^{\bar{z}z}_{N,T}$

$K^{\bar{z}\bar{z}}_{N+1,T} = K^{\bar{z}\bar{z}}_{N,T-1} - K^{\bar{z}x}_{N,T}(K1^{xz}_{N,T})^{-1} K^{z\bar{z}}_{N,T}$

※ LT : Last of time , P : Estimated order

Table 1. The covariance lattice form of EIV for ARMA process

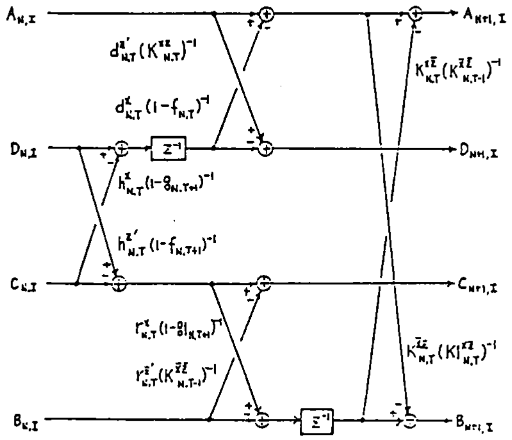

Fig. 2. Lattice form of whitening filter

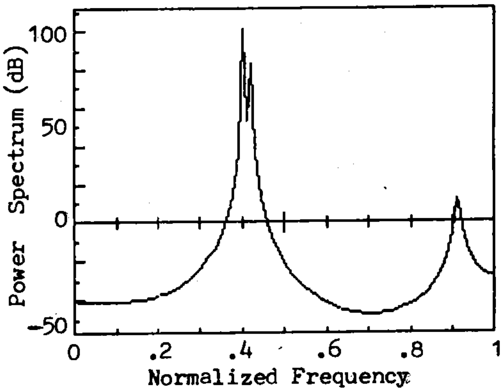

Fig. 5. Estimated spectrum by EIV

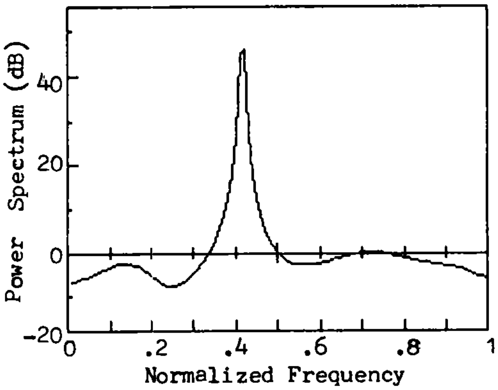

Fig. 3. Estimated spectrum by ELS

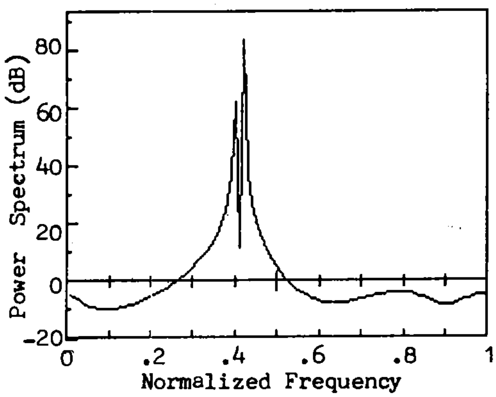

Fig. 6. Estimated spectrum by LEIV

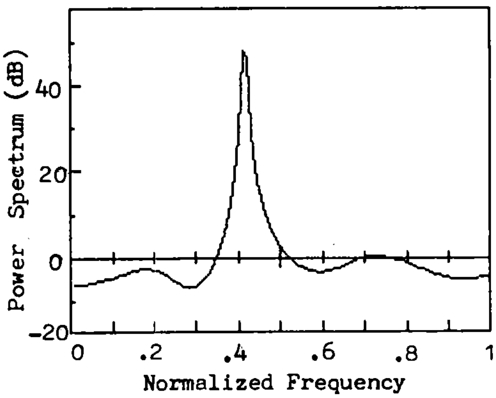

Fig. 4. Estimated spectrum by RML