

Real-Time State Estimation to a Power System

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1. Introduction

On-line real time state estimation is becoming a basic function in modern power system control software and more and more power system control centers have real time state estimation in service. Up to the present, various methods have been proposed for the state estimation in electric power systems. An extensive list of references can be found in the survey paper.(1) Basically, all these papers are based on Weighted Least Squares(WLS) method. When the noise in the measured data possesses certain statistical properties, the resulting estimations are known as unbiased and minimal variance.

Due to non-linear measurements, in the WLS approach, an iterative process based on successive linearization is implied so that good estimates can be obtained. However, it has been shown that by sequential processing of one measurement at a time, estimates of the same accuracy as in the iterative process can be obtained.(2,3) The performance of a WLS estimator may be severely degraded due to the presence of bad data, which may be due to structural error or meter communication failures. The problem of bad data has been considered by using either different performance criterion or certain identification logic based on hypothesis testing procedures.(3-8) The implementation of these methods leads extra computation and storage.

The aim of this paper is to introduce

a real time state estimation algorithm that avoids the difficulties inherent in static state estimation and also avoids any serious attempt to model the time behavior of the system state. In this case of this new estimator algorithm, the Kalman filtering techniques which is known as tracking state estimator and a sparse program techniques are applied.(9) The bad data obtained while measuring the variables of the electric power systems must be rejected before the state estimation is attempted. For the purpose of detection of bad data, a simple test on the residual error is proposed and once the bad data is detected the method developed estimates the size of bad data and implements a correction depending on this size.

The theoretical results are supplemented by digital computer simulation studies with on-line real-time mode.

2. State Estimation of Power System

Let the large scale electrical power system containing N buses and L lines is under normal condition. The state vector X to be obtain is written as follows,

$$X = \begin{bmatrix} \underline{e} \\ \underline{\delta} \end{bmatrix}^T \quad (1)$$

where X : n dimensional vector of the true state. ($n=2N-1$)

\underline{e} : N dimensional voltage magnitude vector.

$\underline{\delta}$: $N-1$ dimensional voltage phase angle vector.

The non-linear equations relating the measurements and the state vectors are

$$\underline{Z} = \underline{h}(\underline{X}) + \underline{V} \quad (2)$$

where \underline{Z} : measurement vector.

$\underline{h}(\underline{X})$: non-linear function.

\underline{V} : error vector.

m : number of measurements.

The optimum state estimate itself is given by

$$\hat{\underline{X}}_{k+1} = \hat{\underline{X}}_k + \underline{P}(\hat{\underline{X}}_k) \underline{H}^t(\hat{\underline{X}}_k) \underline{W}^{-1} [\underline{Z} - \underline{h}(\hat{\underline{X}}_k)] \quad (3)$$

where \underline{H} : Jacobian matrix

\underline{W} : Covariance of error random vector

$$\underline{P}: \underline{H}^t \underline{W}^{-1} \underline{H}$$

We can get a solution by using eq(3), start with some $\hat{\underline{X}}_0$, until $|\hat{\underline{X}}_{k+1} - \hat{\underline{X}}_k| \leq \epsilon$.

2.1 Application of the Kalman Filter

We shall focus our attention on dynamic state estimation theory which is prove to be a better approach to the problem than one already discussed. In obtaining an on-line state estimation algorithm, it is considered the following simple minded model for the time behavior of the system.

$$\underline{X}(k) = \underline{A} \underline{X}(k-1) \quad (4)$$

$$\underline{Z} = \underline{h}(\underline{X}) + \underline{V} \\ = \underline{h}(\underline{X}_0) + \frac{\partial \underline{h}}{\partial \underline{X}} \bigg|_{\underline{X}_0} (\underline{X} - \underline{X}_0) + \underline{V} \quad (5)$$

Here, let $\underline{X} - \underline{X}_0 \rightarrow \underline{\Delta X}$, $\underline{Z} - \underline{h}(\underline{X}_0) \rightarrow \underline{\Delta Z}$

$$\underline{\Delta Z}(k) = \underline{H}(k) \underline{\Delta X}(k) + \underline{V}(k) \quad k=1,2,\dots \quad (6)$$

We can get the following Kalman Filtering recursive equations.

$$\hat{\underline{X}}(k) = \hat{\underline{X}}(k-1) + \underline{K}_k [\underline{Z}(k) - \underline{h}_k(\underline{X}(k-1))] \quad (7)$$

$$\underline{P}(k) = [\underline{I} - \underline{K}_k \underline{H}_k] \underline{P}(k-1) \quad (8)$$

$$\underline{K}(k) = \underline{P}(k-1) \underline{H}_k^t [\underline{\sigma}_k^2 + \underline{H}_k \underline{P}(k-1) \underline{H}_k^t]^{-1} \quad (9)$$

Where $\underline{X}(k)$: state estimate vector after processing k of measurements

$\underline{P}(k)$: covariance matrix

$\underline{Z}(k)$: k -th measurement in \underline{Z}

$\underline{h}(k)$: nonlinear measurement equation for the k -th measurement

$\underline{H}(k)$: linearized measurement equation

$\underline{\sigma}_k^2$: covariance of noise on the k -th measurement in \underline{Z}

\underline{K}_k : gain vector for the k -th measurement

An initial estimate $\hat{\underline{X}}(0)$ and corresponding

covariance matrix $\underline{P}(0)$ are needed to start the computation in the above scheme.

2.2 An Improved Model of Kalman Filter Equation

From here on, the main objective is to express the results of Kalman filter equation in an improved form that will allow us to take advantage of the sparsity involved. Consider the discrete time linear system.

$$\underline{X}(k) = \underline{\Phi}(k, k-1) \underline{X}(k-1) + \underline{\Gamma}(k, k-1) \underline{W}(k-1) \quad (10)$$

$$\underline{Z}(k) = \underline{H}(k) \underline{X}(k) + \underline{V}(k) \quad k=1,2,\dots \quad (11)$$

Here, denote by $\hat{\underline{X}}(k|k)$ the optimal filtered estimate with its corresponding covariance matrix $\underline{P}(k|k)$,

$$\underline{P}(k|k) = E \{ (\hat{\underline{X}}(k|k) - \underline{X}(k)) (\hat{\underline{X}}(k|k) - \underline{X}(k))^t \} \quad (12)$$

Also define $\underline{P}(k|k-1)$ as

$$\underline{P}(k|k-1) = E \{ (\hat{\underline{X}}(k|k-1) - \underline{X}(k)) (\hat{\underline{X}}(k|k-1) - \underline{X}(k))^t \}$$

where $\hat{\underline{X}}(k|k-1) = E \{ \underline{X}(k) | \underline{Z}(k), \dots, \underline{Z}(k-1) \}$

Then the conventional Kalman filter equations are as follows

$$\hat{\underline{X}}(k|k) = \underline{\Phi}(k, k-1) \hat{\underline{X}}(k-1|k-1) + \underline{K}(k) [\underline{Z}(k) - \underline{H}(k) \underline{\Phi}(k, k-1) \hat{\underline{X}}(k-1|k-1)] \quad (13)$$

$$\underline{K}(k) = \underline{P}(k|k-1) \underline{H}^t(k) [\underline{H}(k) \underline{P}(k|k-1) \underline{H}^t(k) + \underline{W}(k)]^{-1}$$

$$\underline{P}(k|k-1) = \underline{\Phi}(k, k-1) \underline{P}(k-1|k-1) \underline{\Phi}^t(k, k-1) +$$

$$\underline{\Gamma}(k, k-1) \underline{Q}(k-1) \underline{\Gamma}^t(k, k-1)$$

$$\underline{P}(k|k) = [\underline{I} - \underline{K}(k) \underline{H}(k)] \underline{P}(k|k-1)$$

By means of various vector-matrix manipulation, equations can be put into a number of equivalent forms. One such equivalent representation involves an alternate expression for the filter gain matrix $\underline{K}(k)$. The final results are as follows

$$\underline{K}(k) = \underline{P}(k|k) \underline{H}^t(k) \underline{R}^{-1}(k) \quad (14)$$

$$\underline{P}(k|k) = [\underline{P}^{-1}(k|k-1) + \underline{H}^t(k) \underline{R}^{-1}(k) \underline{H}(k)]^{-1}$$

Now let $\underline{\Sigma} = \underline{I}$, $\underline{Q} = \underline{Q}$ and define $\underline{F}(k)$, the Fisher information matrix by $\underline{F}(k) = \underline{P}^{-1}(k|k)$

Furthermore, it is considered the case where the measurements are processed sequentially.

3. Bad Data Processing

The WLS estimates due to the bad data are inaccurate and hence need correction. The problem has been considered by several authors. H.Muller has proposed modified performance criteria such as quadratic square root and quadratic constant in combination

with the normal quadratic criterion.(10)
 Instead of modifying the performance criterion function, some authors propose a $J(\hat{X})$ -test in combination with r_w -test(weighted residual test) and r_n -test(normalized residual test). This methods for their implementation requir the evaluation of residues, and sensitivity matrices leading to additional computations.(11,12) Further, some of these methods cannot be applied to deal directly with multiple bad data points. To deal with this contingency, an estimation of the gross error which is called δ -test is proposed recently for bad data process.(13)

From here, we develop a method to detect, estimate and correct the bad data which is done without extra computations available the residual by using the orthogonal transformation matrix when a new measurement is processed.

3.1 Detection of Bad Data

Under the assumption that the measurement error is a normally distributed one with standard deviation σ , and no bad data is present, it is reasonable to assume that the measurement residual is less than or equal to 3σ .(15)

$$Z_{m+1,t} - a_{m+1} \tilde{X} \leq 3\sigma \quad (15)$$

If the increase in J from m -th to $(m+1)$ -th measurement is ΔJ , then

$$\Delta J \leq 3\sigma (a_{m+1} / Z_{m+1,t} - a_{m+1} \tilde{X})^2 \quad (16)$$

Hence, for the purpose of detection, threshold limits can be imposed on either ΔJ or a_{m+1} and the presence of bad data can be detected.

3.2 Estimation of Bad Data

Let $e_{m+1,t}$ is true residual correspond to the true measurement $Z_{m+1,t}$. then

$$e_{m+1,t} = e_{m+1,t} + \alpha = S_{22}(Z_{m+1,t} - a_{m+1} \tilde{X}) + S_{22} \beta \quad (17)$$

where α is the increase in the residual due to the bad data of size β .

$$\beta = e_{m+1,t} (Z_{m+1,t} - a_{m+1} \tilde{X} - 3\sigma) / Z_{m+1,t} - a_{m+1} \tilde{X} \quad (18)$$

3.3 Correction of Bad Data

The modified measurement vector is different from the true measurement vector vector.

$$\hat{Z}_t = Z_t + \delta \quad (19)$$

To obtain Z_t , one has to obtain δ , the correction vector. It may be noted that δ is given by $\delta = S_{12} \beta$

The size β of bad data is already known. Having obtained δ , the correction needed for obtaining \hat{Z}_t can be solved with \hat{Z} replaced by $\hat{Z}_t = \hat{Z} - \delta$. It may be noted that in the detection, estimation and correction scheme proposed, no extra computations are needed except division for the estimation of bad data and a vector-scalar multiplication for the correction. For processing of the next measurement \hat{Z} is replaced by \hat{Z}_t .

4. Experiments

4.1 Computation of \tilde{Y}

The impedance and line charging data for the test model system(5-bus 7-line, 30-bus 41-line, 118-bus 476-line AEP) is given in terms of the line resistance, reactance and charging capacitance per unit (100MVA). R_j and X_j are given as data and the line charging capacitance C_j is normally lumped on the buses at the line terminals with one-half of the total charging of the line at each end.

$$Y_{kk} = G_{kk} + B_{kk} j$$

$$\tilde{Y}_{kk} = Y_{kk} / \tan^{-1}(B_{kk}/G_{kk})$$

where Y_{kk} : Self admittance at node k

$$Y_{km} = G_{km} + B_{km} j$$

$$\tilde{Y}_{km} = -Y_{km} / \tan^{-1}(B_{km}/G_{km})$$

where Y_{km} : Mutual admittance between nodes k and m

Now suppose that bus k has an transformer with nonunity output. If the \tilde{Y} matrix is first computed assuming no transformers are present, then the following scheme should be used to make corrections.

$$Y_{kk,tap} = Y_{kk,notap} + (N_j^2 - 1) Y_j$$

$$Y_{mm,tap} = Y_{mm,notap}$$

where $Y_{kk,notap}$ or $Y_{mm,notap}$ are self admittances computed assuming no tap present in Y_j .

$$Y_{km,tap} = N_j Y_{km,notap}$$

4.2 Experimental Apparatus

Kettering Energy System Laboratory:
 Real-time Power system simulation facility.

- i) Computer Equipment
 - Substation computer(6)
 - Area level computer(1-VAX/750)
 - External-event computer(1)
 - Data-collection computer(2)
 - Digital-relay computers
- ii) Transmission Line Network(TLN)
 - Capable of representing up to 2000 miles of transmission line (π -section) at various voltage(138, 345, 500, 765 KV). M 6800-based impedance relay.
 - Time varying (stochastic) substation quantity.
 - Random load devices.
- iii) Laboratory Peripheral Accelerator
 - Control of analog-to-digital and digital-to-analog converters, digital I/O registers, and real-time clocks.
 - Allows aggregate analog input and output rates up to 150,000 samples per second.
 - Sampling is initiated either by an overflow of the real-time clock or by an externally supplied signal.

4.3 Results and Discussion

The error curve of the results of a computer run based on a set of initial conditions for the estimator are presented in fig. 1. These results are in good agreement with the correct values, and except for a few nodes, the estimated phase angles are to within a degree of their respective corrective values. The total execution time for the 118-bus system was approximately 95 seconds, however

this time can be cut down by using the higher clock rate for starting, stopping, or changing the sample rate at the real-time clock runs on the LPA subsystem. Furthermore, there is still much room for increasing the efficiency of the code.

5 Conclusions

In this study, I have tried to point out some of the major aspects of the real-time state estimation problem and bad data process in power systems. An emphasis was based on examining efficient state estimator algorithms. I have presented some errors of the numerical results of this scheme. Some improvement may result by making a few minor changes in the algorithm. It is prospected that much can be done in the way of improving security and economy in power systems.

References

1. F.C. Schwepper, et al, "Static state estimation in electric power systems", Proc. IEEE vol. 62, pp972-982, July 1974.
2. A.S. Debs, et al, "On-line sequential state estimation for power systems", 4-th PSCC, 1972.
3. H.M. Merrill, et al, "Bad data suppression in power system static state estimation", IEEE Trans. vol. PAS-90, pp 2718-2725, 1971.
4. E. Handschin, et al, "Bad data analysis for power system state estimation", IEEE Trans. vol. PAS-94, pp 329-337, March/April 1975.

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