An Iterative Algorithm for Decentral Stabilization of Large Scale Interconnected System

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1. Introduction

Decentralized control has advantages in saving information link cost and in guaranteeing independent continuous operation of each subsystems even in the case of some accident. But until recently major problem is to stabilizing the overall system with only local feedback.

Some researchers made contributions on this subject. Wang and Davison (1973) introduced fixed mode concept, Corfmot and Morse (1976) have studied strong connectedness concept. Also, Siljak and Vukcevic (1977) made an pole shifting algorithm for designing stable decentralized control using aggregation technique with Lyapunov function. On the other hand, the result of Mahalanabis and Singh (1980) for decentral stabilization was proved to be false by Suh, Moon and Bien (1981).

In this paper an algorithm for designing stable decentralized control is given based on the sufficient condition for stability of large scale interconnected system by Lyou, Kim and Bien (1983) which is modified version of the work by Mahalanabis and Singh (1980).

2. Problem definition

For the large scale interconnected linear time invariant system described below

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{j \neq i} A_{ij} x_j \quad (1)$$

$$, i = 1, \ldots, N$$

the problem is to choose the decentralized state feedback control

$$u_i = K_i x_i \quad , i = 1, \ldots, N \quad (2)$$

so that the overall closed looped system may be stable.

Closed looped system is now as follows.

$$\dot{x}_i = (A_i + B_i K_i) x_i + \sum_{j \neq i} A_{ij} x_j \quad (3)$$

$$, i = 1, \ldots, N$$

It is assumed that the pairs (A_i, B_i), $i = 1, \ldots, N$ are all completely controllable and hence any desired local pole configuration of the subsystem can be obtained by local state feedback (2).

3. A proposition for stability of large scale interconnected system

<u>Proposition 1</u> (Lyou, Kim and Bien)

For the system (3) let $(A_i + B_i K_i)$, $i = 1, \ldots, N$ be asymptotically stable matrices and P_i, $i = 1, \ldots, N$ be positive definite solutions of

$$(A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) = -Q_i$$

$, i = 1, \ldots, N$

where Q_i, $i = 1, \ldots, N$ are arbitrary positive definite matrices. If

$$(A_i+B_iK_i)^T P_i + P_i(A_i+B_iK_i) + 2m_iP_i \quad , i = 1, \ldots, N \tag{5}$$

are all negative definite matrices, then overall system (3) is asymptotically stable. Here m_i, $i = 1, \ldots, N$ are calculated from

$$m_i = \tfrac{1}{2}\sum_{j=1}^{N} d_{ji} \tag{6}$$

$$d_{ii} = \sum_{j\neq i} \left(\max_{\ell}\left(\sum_k \sum_w \left|P_{iilw}A_{ijwk}\right|\right)\right)$$

$$\underset{i\neq j}{d_{ij}} = \max_K\left(\sum_{\ell}\sum_w \left|P_{iilw}A_{ijwk}\right|\right)$$

Proof

See Lyou, Kim and Bien (1983).

Remark 1

m_i, $i = 1, \ldots, N$ are functions of $(A_i + B_iK_i)$, A_{ij}, $j = 1, \ldots, N$, $i\neq j$, P_i for $i = 1, \ldots, N$ respectively. Hence m_i, $i = 1, \ldots, N$ can not be chosen independently of those values.

Remark 2

Once the condition (5) is satisfied, the system (3) is interconnection free stable.

4. An algorithm for designing stable decentralized control

Now go back to the closed looped system (3). We know from Kalman (1960) that if $\max_j(\mathrm{Re}\,\lambda_j(A_i+B_iK_i))$ is less than $-\sigma_i$, then there exists $P_i>0$ such that

$$(A_i+B_iK_i)^T P_i + P_i(A_i+B_iK_i) + 2\sigma_i P_i = -Q_i \tag{7}$$

for any positive definite Q_i.

The converse is true, too.

Also we know that the poles of $A_i+B_iK_i$ can be arbitrarily assigned from the assumption of complete controllability. Hence, if we assigning the poles of $A_i+B_iK_i$, $i = 1, \ldots, N$ so that

$$-\sigma_i = \max_j(\mathrm{Re}\,\lambda_j(A_i+B_iK_i)) \leqslant -m_i \quad , i=1,\cdots,N \tag{8}$$

, then

$$(A_i+B_iK_i)^T P_i + P_i(A_i+B_iK_i) + 2m_iP_i = (-2\sigma_i+2m_i)P_i - Q_i \quad , i=1,\cdots,N \tag{9}$$

become negative definite.

Note that m_i, $i = 1, \ldots, N$ are dependent on pole configuration. Hence, we may not achieve the satisfaction in one step. This leads to iterative algorithm as Siljak and Vukcevic (1977).

Proposition 2

$$\text{If} \quad \sigma_i \geqslant m_i \tag{10}$$

, then closed looped system (3) is asymptotically stable and actually interconnection free stable.

Proof

If a matrix M is positive definite , -M is negative definite. Sum of two negative definite matrix is also negative definite. Hence from (9) $\sigma_i \geqslant m_i$ implies the negative definiteness of LHS of (9).

Remark 3

More strong form can be obtained if (10) is replaced by

$$-Q_i + (2m_i - 2\sigma_i)P_i < 0 \quad , i = 1, \ldots, N$$

But it needs checking negative definiteness of matrices, which is computationally inefficient.

Remark 4

It needs only scalar computation of m_i.

Now, we are in the position of stating the algorithm for designing stable decentralized control.

Step 1 Choose initial σ_i, $i=1,\cdots,N$.

Step 2 Choose decentralized state feedback matrix K_i, $i = 1,\ldots N$ so that the subsystems $(A_i+B_iK_i)$, $i = 1,\ldots,N$ have eigenvalues whose real parts are less than σ_i, $i = 1,\ldots,N$.

Step 3 Solve

$$(A_i+B_iK_i)^T P_i + P_i(A_i+B_iK_i)+2\sigma_i P_i = -Q_i$$

for some positive definite matrices Q_i's.

Step 4 Calculate m_i, $i = 1,\ldots,N$.

Step 5 If $\sigma_i \geqslant m_i$ for all $i = 1,\ldots,N$, then overall system is asymptotically stable and the result is obtained.

If not, let σ_i be larger than m_i (for example $\sigma_i=2m_i$) and go to step 2.

Remak 5

During the procedure of above algorithm, time consuming eigenvalue calculation or definiteness check are not required.

Remark 6

As (10) implies the algorithm gives the information about the amount of pole shifting of each subsystem. This feature has greater advantage over the scheme by Siljak and Vukcevic (1977).

5. Example

The example used by Siljak (1978) is solved with the above proposed algorithm. The result shows that

Siljak and Vukcevic

$K_1 = (93748, 6874, 149)$

$K_2 = (1247, 73)$

$\lambda_1 = -36$, $\lambda_{2,3} = -26\pm 3.5j$, $\lambda_{4,5} = -68.5\pm 6j$

Proposed Algorithm

$K_1 = (4290, 7621, 311)$

$K_2 = (120, 41)$

$\lambda_1 = -25$, $\lambda_{2,3} = -15\pm j$, $\lambda_{4,5} = -11\pm 2.2j$

6. Conclusion

An algorithm for designing decentral stabilizing control is given. The algorithm has computationally efficient over the conventional schemes and has advantages in giving the information about the amount of pole shifting of each subsystems. Thus more smaller local state feedback gain can be obtained.

7. Reference

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