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A Study on the Performance of the Noncoherent FFH-SSMA
Communication Systems over Fading Channels

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ABSTRACT

The performance of noncoherent fast frequency-hopped spread spectrum multiple access communication systems with square-law combining over fading channel is presented. The expression for the probability of error as a performance measure is derived by means of moment-generating function of the decision variables, and the cross-covariance of the fading process the ambiguity function of the transmitted signals.

I. INTRODUCTION

The noncoherent demodulation communication systems with quadratic combining or more practically with square law combining have been shown to be optimum or nearly optimum in fading environment.

Under the ever increasing spread spectrum technique applications such as anti-jam, satellite and cellular mobile communication systems, it is of great interest to study the effects of fading on the performance of spread spectrum communication systems. Gardner et al.[6], and Borth et al.[7] analyzed the effects of fading for direct-sequence systems, and Milstein et al.[8] and Geraniotis et al.[9] for frequency hopping systems with strict assumptions. In this study, the performance of noncoherent fast frequency hopped spread spectrum multiple access(FFH-SSMA) communication systems with square-law combining over dispersive channels is presented in

terms of the probability of error as a performance measure. The channel considered here is selective Rician wide-sense stationary uncorrelated scattering(WSSUS) channel which includes the Rayleigh fading channel as a special case.

II. SYSTEM MODEL

The system to be analyzed is shown in Fig.1. The pseudorandom code generated in the transmitter is synchronized to that in the receiver, but the phase is not.

The Mark and Space matched-filter are matched to mark and space signals transmitted, respectively. The outputs of the matched-filters are squared and summed up with other channels.

In our systems, combining is achieved by accumulating the L successive hops of same information, and two combiner outputs are compared for bit decision.

It is assumed that the frequency sepa-

ration of the Mark and Space signals is taken to be $1/T$ where T is the symbol duration.

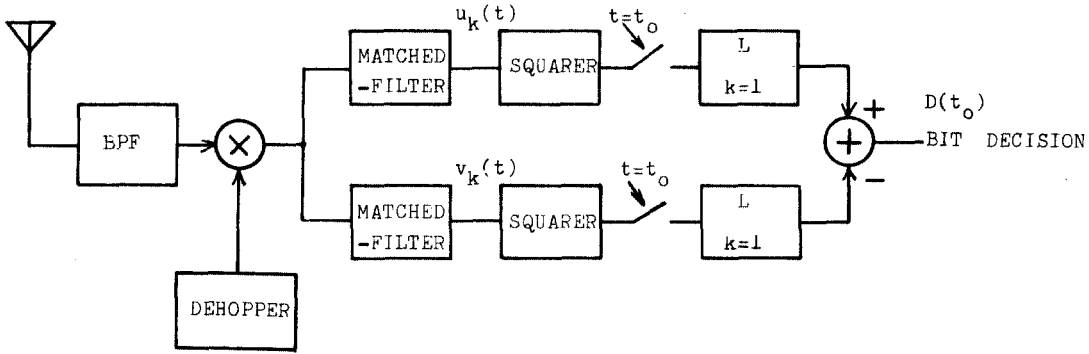


Fig.1. Noncoherent FFH-SS System Model

III. PROBABILITY OF ERROR WITH NO HITS

Let the complex envelope of Mark and Space signals in the i th user transmitter denote by

$$S_i(t) = \sqrt{\frac{2E}{T}} \exp\{-j(2d_i - 1)\frac{\pi}{T}t + \theta_{d_i}\} \quad 0 \leq t < T \quad (3-1)$$

where E and d_i are the energy of real binary waveform, the data of the i th user in the interval $0 \leq t < T$, respectively, and θ_{d_i} is the relative phase of the transmitted signal and is a uniformly distributed random variable with constant over $0 \leq t < T$.

The decision variable in the k th hop is $d_k(t_0)$ where

$$d_k(t_0) = |u_k(t_0)|^2 - |v_k(t_0)|^2 \quad (3-2)$$

In matrix notation,

$$d_k(t_0) = Z_k^{i*} Q Z_k \quad (3-3)$$

where $'$, $*$ represent the transpose and

complex conjugate of the matrix, and

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (3-4)$$

After accumulating the L successive hops, the decision variable becomes

$$D(t_0) = \sum_{k=1}^L Z_k^{i*} Q Z_k \quad (3-5)$$

The probability of error when the Mark is transmitted, is then

$$P_{L,M} = \Pr\{D(t_0) < 0 \text{ /mark}\} \quad (3-6)$$

To derive the desired probability distribution of the random variable $D(t_0)$, we first obtain the moment-generating function of $D(t_0)$

$$M_D(s) = E\{\exp(sD) \text{ /mark}\} \quad (3-7)$$

It can be shown that[5]

$$M_D(s) = \prod_{k=1}^L \frac{\exp\{-Z_k^{i*} K_Z [I - (I - sK_Z Q)^{-1} Z_k]\}}{|I - sK_Z Q|} \quad (3-8)$$

where

$$\bar{Z}_k = E\{Z_k\} = \begin{bmatrix} \bar{u}_k \\ \bar{v}_k \end{bmatrix} \dots\dots (3-9)$$

and

$$K_Z = E\{(Z_k - \bar{Z}_k)(Z_k - \bar{Z}_k)^{*}\} \\ = \begin{bmatrix} k_{uu} & k_{uv} \\ k_{uv}^* & k_{vv} \end{bmatrix} \dots\dots (3-10)$$

From the text[5], the mean-square value of the random component of the fading process which is a zero-mean Gaussian random process, is

$$k = \int |W(\tau, \mu)|^2 V_{\eta}(\tau, \mu) d\tau d\mu \dots (3-11)$$

where $V_{\eta}(\dots)$ is the scattering function of the channel and

$$W(\tau, \mu) = \int h(t_0 - \tau - x) S(x) e^{-j\mu x} dx \dots\dots\dots (3-12)$$

where $h(\dots)$ is the equivalent low-pass impulse response of the receiver filter. For matched-filter receiver,

$$h(t) = S(t_0 - t) \dots\dots\dots (3-13)$$

Then Eq.(3-12) represents the ambiguity function of the transmitted signals in radar terminology.

It follows that[2] the decision variable $D(t_0)$ is distributed as

$$D = \frac{1}{2} [e_1 X_n^2(2L, a_1) + e_2 X_n^2(2L, a_2)] \dots\dots\dots (3-14)$$

where e_i , $i=1,2$ is the eigenvalue of $K_Z Q$ and $X_n^2(2L, a_i)$, $i=1,2$ is statistically independent, non-central chi-square random variates with $2L$ degrees of freedom, and non-central parameter a_i . The expression equivalent to Eq.(3-6) is

$$P_{L,M} = \Pr\left\{ \frac{X_n^2(2L, a_1)}{X_n^2(2L, a_2)} < \frac{|e_2|}{e_1} \right\} \dots\dots\dots (3-15)$$

and consequently

$$P_{L,M} = 1 - Q(b_1, b_2) \frac{\exp\{-(b_1^2 + b_2^2)/2\}}{(1+b_3)^{2L-1}} \sum_{m=0}^{L-1} C_m I_m(b_1 b_2) \dots\dots (3-16)$$

where $Q(a,b)$ is the Marcum's Q-function and $I_m(a)$ is the m th order modified Bessel function of the first kind and

$$C_m = \sum_{k=m}^{L-1} \binom{2L-1}{k-m} \left(\frac{b_1}{b_2 b_3}\right)^m b_3^k (1 - \delta_{m0}) \dots\dots (3-17)$$

$$\text{with } b_1 = \sqrt{\frac{a_1 b_3}{1+b_3}}$$

$$b_2 = \sqrt{\frac{a_2}{1+b_3}}$$

$$b_3 = \frac{e_1}{|e_2|} \dots\dots (3-18)$$

IV. PERFORMANCE OVER WSSUS CHANNEL

For the WSSUS fading channel,

$$\frac{1}{2} E\{h(t, \tau) h^*(t-s, \sigma)\} = \rho(t-s, \tau) \int (\tau - \sigma) = r(\tau) g(t-s) \dots\dots (4-1)$$

where $h(\dots)$ represents a time-varying equivalent low-pass impulse response of the channel. As usual[1,5] the scattering function which is the Fourier transform of $\rho(\tau, t-s)$ with respect to $t-s$, is assumed to be a Gaussian shaped, that is,

$$V_T(\tau, \mu) = 2\sigma^2 \exp[-4\tau^2/B_D^2 + \mu^2/B_C^2] \dots (4-2)$$

where σ^2 is the average power that would be received when a sinusoid of unity amplitude is transmitted and B_D and B_C are the Doppler spread bandwidth, the correlation bandwidth, respectively, which are defined as a frequency spacing between the 1/e points on the respective curve.

Confining the intersymbol interference due to frequency-selective effect to the two adjacent data bits, the probability of error conditioned on no hits, θ_d is the average probability of error over four possible cases, that is,

$$P_{L,M}(\text{error/no hits}, \theta_d) = \frac{1}{4} \sum_{d_{-1}d_{+1}} P_{L,M}(\text{error/no hits}, \theta_d, d_{-1}, d_{+1}) \dots (4-3)$$

An individual probability of error in the above equation corresponds to a conditional ambiguity function on the two adjacent data bits

$$W_{i,j}(\tau, \mu) = \int_{d_{-1}d_{+1}}^* S_{d_{-1}d_{+1}}(t) S_{i,j}(t+\tau) \times \exp(-j\mu t) dt \quad (4-4)$$

The corresponding element in Eq.(3-10) is

$$k_{i,j} = \delta(w_{-1}, w_0) k_{i,j}(\tau > 0) \delta(w_0, w_{+1}) k_{i,j}(\tau < 0) \dots (4-5)$$

where w_{-1}, w_0 , and w_{+1} are the hopping frequencies of the 3 successive hops. In effect, $\delta(\dots)$ represents the action of dehopper on the signal.

If we take the average for the expression given in Eq.(4-3) assuming the random

hopping pattern, we can get

$$P_{L,M}(\text{error/no hits}, \theta) = \left(\frac{N-2}{N}\right)^2 P_{L,M}(\theta, w_{-1} \neq w_0 \neq w_{+1}) + \frac{2N-4}{N^2} [P_{L,M}(\theta, w_{-1} = w_0 \neq w_{+1}) + P_{L,M}(\theta, w_{-1} \neq w_0 = w_{+1})] + \frac{4}{N^2} P_{L,M}(\theta, w_{-1} = w_0 = w_{+1}) \quad (4-6)$$

where N is the number of frequencies over which the signal can hop.

Averaging the Eq.(4-6) over the random variable θ results in

$$P_L = P_{L,M} = \frac{1}{2\pi} \int_0^{2\pi} P_{L,M}(\text{error}/\theta) d\theta \dots (4-7)$$

For first-order Markov hopping patterns, the probability of hit from the other user is, for specular component, $2/(N-1)$ and, for fading component, upper bounded $4/N$ [9]. If the patterns are mutually independent each other and there are total K users in the channel, the probability of no hits from the K-1 signals is

$$P(\text{no hits}) = (1 - P_h)^{K-1} \dots (4-8)$$

where P_h is the probability of a hit from a given signal, and is bounded as

$$P_h \leq \frac{2}{N-1} + \frac{4}{N} \dots (4-9)$$

Even when the hits occur, Eq.(3-16) is also applicable so that no further discussions are made.

V. CONCLUSIONS

Using Eq.(3-16), numerical results are obtained for several classes of the fading

channel, i.e., for

- 1) nonselective channels
- 2) frequency-selective channels
- 3) time-selective channels
- 4) doubly-dispersive channels

with the special cases of Rayleigh

The expressions derived are quite useful for the analysis of the system in jamming environment.

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