NUMERICAL FLOOD ROUTING METHOD

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I. Introduction

Flood routing methods are described as belonging to either one of two classes : (1) reservoir routing ; and (2) stream channel routing. The motions of flood wave velocity and attention for each of these two classes are introduced.

A general classification of flood routing methods is attempted on the basis of the following criteria :
(1) the equations used in the formulation ;
(2) the overall approach to data collection ; and (3) the solution technique.

Reservoir and stream channel routing are described in detail, with particular emphasis on the physical processes involved. In stream channel routing, the following three approaches are recognized : (1) the classical approach, of which the Muskingum method is a notable example ; (2) the numerical approach, based on the numerical solution of the complete Saint Venant equations, either by characteristic of finite difference methods ; and (3) the simplified approach, which uses a convection-diffusion equation to describe flood wave movement. A closing remark focuses attention on the unified theory of flood wave movement in terms of kinematic, diffusive and dynamic waves.

Classification of Flood Routing Methods

Based on Equations used
- Mass-balance : Storage equation and an auxiliary storage-outflow relationship.
- Mass-and-momentum-balance : Saint Venant equations(

Based on Approach to Data collection:
- Hydrologic Routing : Observations for channel reaches.
- Hydraulic Routing : Measurements of channel Characteristics at individual cross-sections.

Based on Solution Technique:
- Analytical Routing : Differential equations ; continuous domain.
- Numerical Routing : Algebraic equations ; discrete domain.

II. The Numerical Approach

a. The Numerical Solution of the Saint Venant

T e numerical solution of the Saint Venant equations can be carried out by either : (1) the method of characteristeristics ; or (2) finite difference methods. In the method of characteristics, the two partial differential equations (water continuity and motion) are replaced by four ordinary differential equations which are solved numerically on a characteristic grid. The intersections of characteristic lines on the x-t plane define the characteristic grid.

In the finite difference methods, the functions(e,g., discharge Q, flow area A, stage Y) and their derivatives (e.g., $\partial Q/\partial x$, $\partial A/\partial t$) are expressed in terms of their values on a rectangular grid defined on the x-t plane. A finite difference scheme is a formula expressing a relationship between neighboring values on the rectangular grid. There are two types of finite difference Schemes : (1) explicit ; and (2) implicit. Explicit schemes are those that advance the solution in times and space by solving for the unknown variables at a number of grid points.

Explicit schemes are relatively simple to formulate, but are usually limited to a small time step Δt by considerations of numerical stability. Implicit schemes require the inversion of a matrix, but are not subject to the strict stability criterion of explicit schemes. In general, implicit schemes are more efficient than explicit schemes in their use of computational resources.

b. The Muskingum-Cunge Method

The Muskingum-Cunge method of stream channel routing is a variation due to Cunge of the classical Muskingum method. It is based on the realization that a four-point numerical analog of the kinematic wave equation and the Muskingum storage relationship lead to the same routing equation.

In effect, the kinematic wave equation can be written as follows:

$$\frac{\partial Q}{\partial t} + C\,\frac{\partial Q}{\partial x} = 0 \quad \cdots (1)$$

where Q : the flood wave discharge

C : constant

Equation (1) is discretized on the x-t plane.

$$\frac{x(Q_j^{n+1}-Q_j^{n})+(1-x)(Q_{j+1}^{n+1}-Q_{j+1}^{n})}{\Delta t} + C\,\frac{Y(Q_{j+1}^{n}-Q_j^{n})+(1-Y)(Q_{j+1}^{n+1}-Q_j^{n+1})}{\Delta x} = 0 \cdots (2)$$

in which x and Y = weighting factors ; Δx=spaee interval ; Δt= time interval ; c=constant.

setting Y=0.5, Eq. (2) can be expressed as :

$$\frac{Q_j^{n}+Q_j^{n+1}}{2} - \frac{Q_{j+1}^{n}+Q_{j+1}^{n+1}}{2} = \frac{\frac{\Delta x}{c}[xQ_j^{n+1}+(1-x)Q_{j+1}^{n+1}] - \frac{\Delta x}{c}[xQ_j^{n}+(1-x)Q_{j+1}^{n}]}{\Delta t} \cdots (3)$$

In one time increment, the left-hand side of Eq. (3) is I–O ; the right-hand side is dv/dt if

$$V = \frac{\Delta x}{c}[xI + (1-x)O] \quad \cdots (4)$$

provided $K = \Delta x/c$

The parameter x is recognized as a weighting factor.

Cunge derived the numerical diffusion coefficient μ_n of the discretized kinematic wave equation as

$$\mu_n = C\Delta X\left(\frac{1}{2} - X\right) \quad \cdots (5)$$

by matching this diffusion coefficient with the physical diffusion coefficient of the convection-diffusion equation, the following expression for x is obtained :

$$X = \frac{1}{2}\left(1 - \frac{q_0}{S_0 C \Delta X}\right) \quad \cdots (6)$$

By defining the courant number C as the following ratio of celerities :

$$C = \frac{c}{\left(\frac{\Delta X}{\Delta t}\right)} = c\frac{\Delta t}{\Delta X} \quad \cdots (7)$$

and the cell Reynolds number D as the following ratio of diffusivities :

$$D = \left(\frac{q_0}{2S_0}\right) \Big/ \left(\frac{C\Delta X}{2}\right) = \frac{q_0}{S_0 C \Delta X} \quad \cdots (9)$$

the coefficients Co to C_3 can be expressed in the following reduced

$$C_o = 1+C+D \qquad (9)$$

$$C_1 = (1 + C - D)/C_o \qquad (10)$$

$$C_2 = (-1 + C + D)/C_o \qquad (11)$$

$$C_3 = (1 - C + D)/C_o \qquad (12)$$

Depending on the modeling needs and resources, the calculation of the parameters in the Muskingum-Cunge method can proceed in one of two ways : either by using (a) constant parameters, or (b) variable parameters.

The essential difference between the two is linearity, while this is not the case in computations using variable parameters.

c. Kinematic Wave Modeling Techniques

The kinematic wave belongs to a class of wave motions in which the wave property follows from the equation of continuity alone kinematic waves exist if there is some sort of functional relationship between the discharge, Q, the quantity of water stored per unit distance, and the position, X.

properties of kinematic wave mathematical model. The waves will be considered only for one-dimensional systems.

The continuity equation,

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial X} - q(x,t) = 0 \quad \text{-- (13)}$$

where A is the cross-sectional flow area, Q is the discharge and $q(x,t)$ is the lateral inflow per unit length.

The second equation required is the realtionship

$$Q = Q(h,x) \quad \text{--- (14)}$$

This equation states that there is a unique functional relationship between the discharge and the stage, h, at every position X. For Prismatic channels Equation (14) is equivalent to equating the friction slope S_f to the bed slope, So.

Chezy equation $Q = CA\sqrt{RSo}$ --(15)

C : chezy resistance coefficient

R : radius

in the manning formula

$$Q = \frac{1.49}{n} AR^{2/3} S_o^{1/2} \quad \text{--- (16)}$$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial Q}\frac{\partial Q}{\partial t} = \frac{1}{C}\frac{\partial Q}{\partial t} \quad \text{--- (17)}$$

$$\frac{1}{c}\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial X} = q(x,t) \cdots (18)$$

$$\frac{dQ}{dx} = q(x,t) \quad \text{--- (19)}$$

$$\frac{dx}{dt} = c = \left(\frac{\partial Q}{\partial A}\right)_{x = \text{constant}} \cdots (20)$$

Eq (19), (20) are known as the characteristic equations.

If there is no lateral inflow, Q is a constant along the characteristic curves given by Eq (20). C is known as the celerity of the kinematic wave.

in eq (13) $Q = CB\sqrt{So}\, h^{3/2}$ (21)

From Eq (19) $\frac{dQ}{dx} = q(x,t) = 0$ --- (22) $Q = \text{Constant}$ ...(23)

$$\frac{dx}{dt} = \left(\frac{\partial Q}{\partial A}\right) = \frac{1}{B}\frac{\partial Q}{\partial h} = \frac{3}{2}C\sqrt{S_o h} \quad \text{--- (24)}$$

If Q is constant along the characteristic then h is also constant, so the characteristics are straight lines.