MATHEMATICAL SIMULATION MODEL OF FLOW INDUCED CIRCULATION IN A HARBOR

Tae Hoon Yoon
Professor
Department of Civil Engineering
Hanyang University
Seoul, Korea

Sung Bum Yoon
Research Assistant
Department of Civil Engineering
Hanyang University
Seoul, Korea

ABSTRACT

The formulation of depth-averaged two-dimensional mathematical model for the analysis of tide induced circulation in a harbor by the Galerkin finite element technique is presented. In integration of the Galerkin approach in time both explicit and implicit method have been tested for one and two dimentional water bodies, and the two step Lax-Wendroff explicit method is found to be effective than the implicit in reducing computing time. The essential characteristics of the tide induced flow in Busan Harbor with two open boundaries has been foccud to be reproduceable in the numerical model and the simulated results encourage that the model can be used as a predictive tool.

1. INTRODUCTION

In growing number of problems such as water quality, maintaining waterways, dumping wastes, and dredging in estuaries, bays, and harbors, the importance of knowledge about the behavior of circulation induced by tide has been stressed, and many mathematical models about circulation and dispersion have been proposed. The major impetus has been stemmed from the necessity which is about the detailed studies of circulation and dispersion and its development of transient predictive model.

Numberical models dealing circulation in estuarine water bodies have been implemented by finite difference scheme. (2,6,7,13) But recently more attention is paid on the finite element method due to the advantage of flexbility to accomodate the irregular geometry and the topography of bottom. (3,10,11, 15,18,19,20) The advantage, however, has to suffer from numerical integration in time. For the numerical integration in this shallow water problems, the two-step explicit Lax-Wendroff scheme(17) is mainly used, which has been found most suitable.(10,11)

The model described herein predicts vertically integrated longitudinal and lateral velocity pattern as well as tidal elevation distribution. The validity of the numerical model has been checked by comparing the computed results in one dimensional water body with known analytical solution. The performance of the model also has been tested by comparing the simulated results with the observed values.

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2. GOVERNING EQUATIONS

The equations of motion for a two dimensional flow known as the shallow water equation can be obtained by integrating the Reynold equations over the depth. Assuming that the vertical accelerations are negligible compared to gravity, i.e., pressure distribution is hydrostatic and the fluid is well mixed, the continuity and horizontal equations become(9,10)

$$\frac{\partial \eta}{\partial t} + \{(h+\eta)u_i\}_{,i} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j u_{i,j} + g\eta_{,j} + \tau_{Bi} = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which u_i denotes depth-average velocity in i-direction, η is water elevation measured from the undisturbed water level and h is water depth from the said level, τ_{Bi} represents the shear stress at bed in i-direction. For the simplification of the model the effects of Coriolis force, atmospheric pressure difference, and horizontal viscosity terms are ignored. Throughout this paper index notation is used, for instance, partial differentiation with respect to x_i is denoted by ($)_{,i}$. The summation is represented by repeated indices.(4)

To solve the system of eqs.(1) and (2) boundary conditons are needed to be established. In this study the boundary consists of three parts:

on land type boundaries

$$u_i n_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

if a river enters the body of water, velocity is prescribed

$$u_i = \bar{u}_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

water elevation is specified on the open sea boundary

$$\eta = \bar{\eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where the overbar denotes the prescribed value on the boundary and η_i represents the component of vector normal to the boundary. As an initial conditions, the initial state has to be given or assumed.

3. FINITE ELEMENT FORMULATION

In order to build finite element model the monentum equation (2) and continuity equation (1) can be expressed in the following Galerkin weighted residual method.

$$\int_V \left(u_i^* \frac{\partial u_i}{\partial t}\right) dV + \int_V (u_i^* u_j u_{i,j}) dV + \int_V (u_i^* g\eta_{,i}) dV + \int_V (u_i \tau_{Bi}) dV = 0 \quad ..(6)$$

$$\int_V \left(\eta^* \frac{\partial \eta}{\partial t}\right) dV + \int_V \left[\eta^* \{(h+\eta)u_i\}_{,i}\right] dV = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where u_i^* and η^* are the weighting functions for velocity and water elevation, respectively. The flow domain of interest is divided into small regions called finite elements. If the same interpolation function is used for both velocity and water elevation, then approximate equations for trial function in each triangular element can be written as

$$u_i = \phi_\alpha u_{\alpha i}, \qquad \eta = \phi_\alpha \eta_\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

in which $u_{\alpha i}$ denotes the nodal values of velocity at node α in the i - direction and η_α represents water elevation at node α. By the same fashion approximate equations for the weighting function can be given as

$$u_i^* = \phi_\alpha u_{\alpha i}^* \ , \quad \eta^* = \phi_\alpha \eta_\alpha^* \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

in which $u_{\alpha i}^*$ and η_α^* are the corresponding nodal values of the weighting functions. Substituting eqs.(8) and(9) into eqs.(6) and(7) and rearranging the terms, the finite element governing equations can be obtained for all arbitrary nodal values of velocity and water elevation. The procedure leading to eqs. (10) and (11) has been worked out by kawahara[11] and used here.

$$A_{\alpha i\beta j}\dot{u}_{\beta j} + B_{\alpha i\beta j\gamma k}u_{\beta j}u_{\gamma k} + C_{\alpha i\beta}\eta_\beta + D_{\alpha i} = 0 \quad \ldots\ldots\ldots\ldots \quad (10)$$

$$E_{\alpha\beta}\dot{\eta}_\beta + G_{\alpha\beta j\gamma}(h_\gamma + \eta_\gamma)\ u_{\beta j} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where

$$A_{\alpha i\beta j} = \int_v (\phi_\alpha \phi_\beta)\ \delta_{ij} dV$$

$$B_{\alpha i\beta j\gamma k} = \int_v (\phi_\alpha \phi_\beta \phi_{\gamma,j})\ \delta_{ik} dV$$

$$C_{\alpha i\beta} = g\int_v (\phi_\alpha \phi_{\beta,i}) dV$$

$$D_{\alpha i} = \int_v (\phi_\alpha \tau_{Bi}) dV$$

$$E_{\alpha\beta} = \int_v (\phi_\alpha \phi_\beta) dV$$

$$D_{\alpha\beta j\gamma} = \int_v \{\phi_\alpha (\phi_\beta \phi_\gamma)_{,j}\} dV$$

The above expressions for each finite element can be assembled for the whole flow domain to yield the following equation :

$$M_{\alpha\beta}\dot{v}_\beta + K_{\alpha\beta}v_\beta + F_\alpha = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where v_β denotes the unknown variables u_β and η_β together. $M_{\alpha\beta}$ is made up of $A_{\alpha\beta}$ and $E_{\alpha\beta}$ and $K_{\alpha\beta}$ contains $B_{\alpha i\beta j\gamma k}u_{\gamma k}$ and $G_{\alpha\beta j\gamma}\ u_{\beta j}$. All other terms are included in F_α.

4. NUMERICAL INTEGRATION IN TIME

Two different methods, Crank - Nicolson implicit scheme[14,16] and two-step Lax-Wendroff explicit scheme,[10,11] were applied in this paper. During short time increment Δt, assume that

$$\left(\frac{\partial v_\beta}{\partial t}\right)^{n+\frac{1}{2}} \doteqdot \frac{v_\beta^{n+1} - v_\beta^n}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$v_\beta^{n+\frac{1}{2}} \doteqdot \frac{v_\beta^{n+1} + v_\beta^n}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

where superscript n denotes the time step. Introducing these results into eq.(12) the following Crank-Nicolson implicit scheme is derived

$$\left(\frac{2}{\Delta t}M_{\alpha\beta} + K_{\alpha\beta}\right)^{n+\frac{1}{2}} v_\beta^{n+1} = \left(\frac{2}{\Delta t}M_{\alpha\beta} - K_{\alpha\beta}\right)^{n+\frac{1}{2}} v_\beta^n - (F_\alpha^{n+1} + F_\alpha^n) \quad \ldots\ldots\ldots \quad (15)$$

To solve the non-linear eq.(15) the conventional iterative method within each time step is employed here. For explicit method, two-step Lax-Wendroff scheme is used.

$$M_{\alpha\beta}v_\beta^{n+\frac{1}{2}} = M_{\alpha\beta}v_\beta^n - \frac{\Delta t}{2}(K_{\alpha\beta}v_\beta^n + F_\alpha^n) \quad \ldots\ldots (16)$$

$$M_{\alpha\beta}v_\beta^{n+1} = M_{\alpha\beta}v_\beta^n - \Delta t\,(K_{\alpha\beta}v_\beta^{n+\frac{1}{2}} + F_\alpha^{n+\frac{1}{2}}) \quad \ldots\ldots (17)$$

To avoid the inversion of mass matrix $M_{\alpha\beta}$, the diagonal lumped matrix [1,21] is used.

$$L_{\alpha\beta}v_\beta^{n+\frac{1}{2}} = M_{\alpha\beta}v_\beta^n - \frac{\Delta t}{2}(K_{\alpha\beta}v_\beta^n + F_\alpha^n) \quad \ldots\ldots (18)$$

$$L_{\alpha\beta}v_\beta^{n+1} = M_{\alpha\beta}v_\beta^n - \Delta t\,(K_{\alpha\beta}v_\beta^{n+\frac{1}{2}} + F_\alpha^{n+\frac{1}{2}}) \quad \ldots\ldots (19)$$

where L denotes the lumped mass matrix.

The criteria used to determine the time step were as follows:

for implicit scheme[15] $\Delta t \leq \frac{T}{20}$ ...... (20)

for explicit scheme[5,20] $\Delta t \leq \min\left(\frac{\Delta x}{\sqrt{2}\sqrt{gh}}\right)$ ...... (21)

where T is period of tide and Δx is side length of element

5. VERIFICATION AND APPLICATION OF MODEL

As the first test for verification of the model, a rectangular basin is used since its analytical solution has been derived.[9,12,15,19,20] One of the four sides of the rectangular is open boundary and three other sides are land boundaries. At the open boundary water elevation is prescribed and along the remaining boundaries zero normal velocity condition is maintained at each time step. The computation has been carried out by both implicit and explicit method and the numerical results of both implicit and explicit methods show good agreement to the analytical solution except for the superimposed oscillation as is often the case in implicit method.[15]

As the second test, the model is applied to L-shaped two dimensional water body with narrow one side of open boundary. Both results show reasonably good agreement, but the implicit sheeme takes more time than the explicit one as is done in the rectangular case. From the computational point of view, using explicit scheme is found to be desirable because of less computational time.

The application of the model to actual situation is for Busan Harbor which is located at the South-east of Korea and has serious water pollution problem due to ever increasing inflow of pollutants. Fig. 1 shows the finite

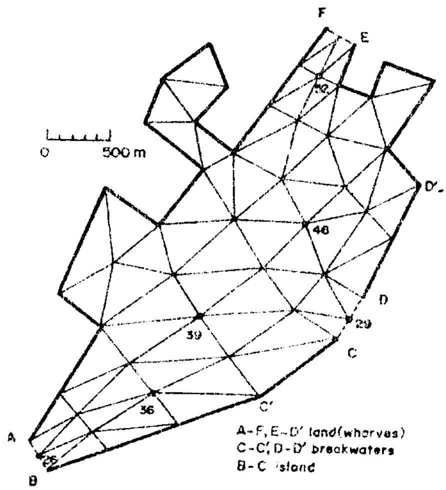

Fig.1 Finite element grid of Busan Harbor

element idealization of Busan Harbor based on the three-node triangular finite element. For the numerical integration in time, the explicit two-step Lax-Wendroff scheme is used owing to the aforementioned reasonings throughout the applications. There are two open boundaries (A-B, C-D) in this domain of which A-B is regarded as free boundary(11). It means that the flow passes through the boundary back and forth.

To investigate the nature of the free boundary, a simplified configuration to Busan Harbor as shown in Fig.2 is employed.

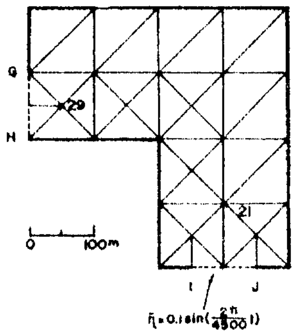

Fig.2 Finite element grid of simplified Busan Harbor

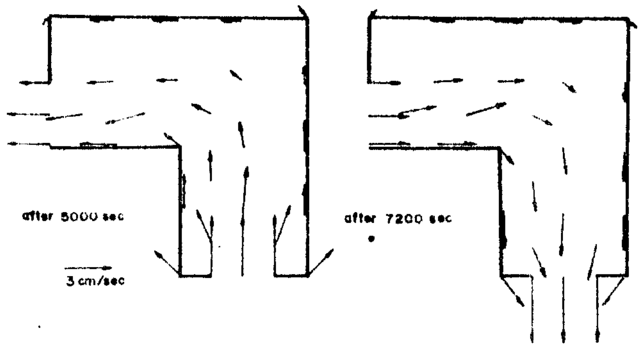

Fig.3 Computed velocity field (one cycle of 4500 sec)

On the free boundary (G-H), no boundary condition is imposed on velocity and water elevation as well. At the open sea boundary (I-J), water elevation is specified and along the remaining boundaries zero normal velocity of land boundary condition is maintained. The computed flow patterns shown in Fig.3 explain the free boundary. Plotted in Fig.4 are the computed velocity and water elevation at nodes 21 and 29. The results of the test of the simplified

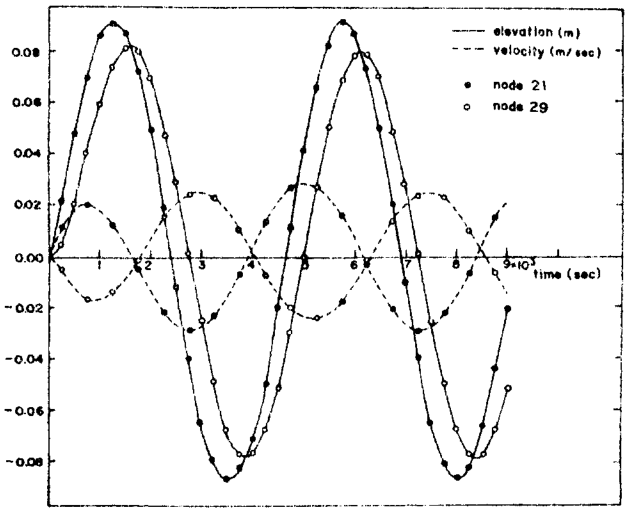

Fig.4 Computed velocities and water elevations

harbor encourage that the model can be applied to the actual geometry of Busan Harbor.

Boundary conditions for the actual Busan Harbor:

on the free boundary (A-B), no boundary conditions is imposed,

on the open sea boundary (C-D),

$$\bar{\eta} = 0.6\ (1-\cos \frac{2\pi}{T} t) \qquad \text{.............................} \quad (22)$$

on the river inflow boundary (E-F),

$$\bar{u}_n = -0.05 \qquad \text{...} \quad (23)$$

and along the remaining land boundaries,

$$\bar{u}_n = 0 \qquad \text{..} \quad (24)$$

where T is the period of tide (44700 sec) and $\bar{u}_n$ is normal velocity measured by m/sec. As the initial conditions, still water condition (cold start) is assumed.

In the determination of time step eq.(21) is used and water depth h in the case of Busan Harbor is measured from mean low-water spring level (MLWS).

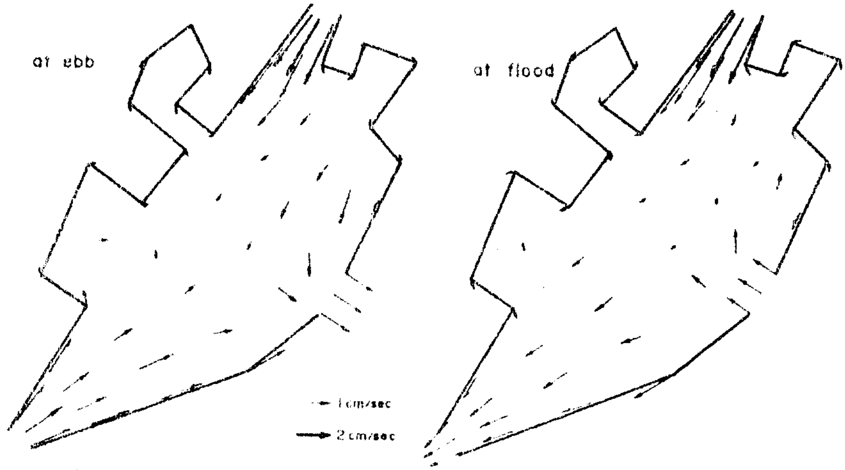

Fig.5 Simulated tidal currents in Busan Harbor

Fig.5 illustrates the computed currents at flood and ebb. The computed water elevations and current velocities at representative nodes are shown in Fig.6 The simulated current patterns are well in agreement to the measurements. The computed velocities are generally less than the observed[8]. This seems due to the fact that the observed velocities are generally the maximum values and the computed velocities are the depth averaged one.

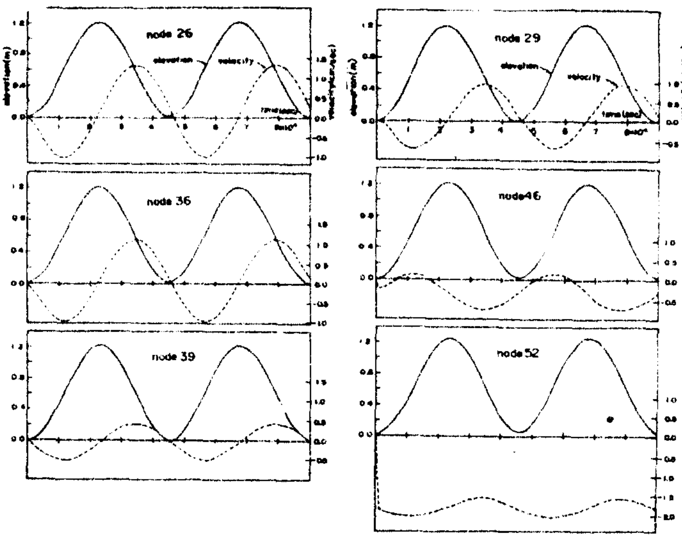

Fig.6 Simulated tidal velocities and tidal elevations at representative points in Busan Harbor

6. CONCLUSIONS

Depth-averaged two-dimensional numerical model capable of predicting the tidal characteristics with two open sea boundaries is presented. For integration of the Galerkin finite element technique in time, some of explicit and implicit methods have been tested. From the resulting tests of the model for one and two dimensional water bodies, the two step Lax-Wendroff explicit method is found to have advantages over the implicit method in reducing the computing time. The flow field of a water body with a free and an open boundary, which is similar to Busan Harbor in character, has been investigated by the numerical model. Then, the numerical model has been applied to Busan Harbor and found that the essential tidal flow characteristics can be reproduced in the model reasonably well.

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