Critical Review of Reconstruction Filters for Convolution Algorithm

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ABSTRACT

The Fourier convolution algorithms are used to reconstruct a 3-D density function from the projection data sets. The convolved data are then back projected to obtain a density function. There are several choices of the weighting function for the design of the reconstruction(deblurring) filter.

Present paper reviews the design of reconstruction filter considering the problems such as the effects of sampling rate, aliasing, and noise.

In computerized tomography, X-ray transmission measurements are recorded on a computer memory and a mathematical algorithm is applied. This produces a numerical discription of the tissue density of a thin slice of the body.

The Fourier convolution algorithm for a 2-D density function f(x,y) uses the following equation,

$$f(x,y) = \frac{1}{(2\pi)^2}\int_0^{\pi} d\theta \int_{-\infty}^{\infty} d\omega \; |\omega| P(\omega,\theta) \; \exp\left\{ i\omega(x \cos\theta + y \sin\theta)\right\} \quad (1)$$

where ω is the spatial frequency and $P(\omega,\theta)$ is the Fourier transform of the projection data. Since the inverse Fourier transform of $|\omega|$ does not converge, the convolution theorem applied to the inner integral in Eq.(1) is valid only in a limiting case. Thus $|\omega|$ is replaced with a function $\Phi(\omega)$ which approximates $|\omega|$ for $|\omega|<\omega_c$ and smoothly approaches to zero for $|\omega|>\omega_c$.

A direct convolution algorithm developed first by Ramachandran and Lakshiminarayanan adopted a particular choice of weighting function. If $\varphi(t)$ is the inverse Fourier transform of $\Phi(\omega)$, it is defined as,

$$\varphi(0) = \frac{\pi}{2a^2}$$

$$\varphi(ka) = -\frac{2}{\pi k^2 a^2}\ , \text{ for } k \text{ odd integer}$$

$$= 0 \qquad , \text{ for } k \text{ even integer}$$

$$\Phi(\omega) = |\omega| \,\text{sinc}^2(\tfrac{\omega a}{2}), \quad \text{for } |\omega| \leq \tfrac{\pi}{a}$$

where sinc x = sin x/x. For all range of ω, $\Phi(\omega)$ is a product of a periodic triangular function with period $2\pi/a$ and the function $\text{sinc}^2(\omega a/2)$. The function $\text{sinc}^2(\omega a/2)$ results from the linear interpolation between samples in the backprojection. The reconstruction response of the Ram and Lak filter was shown to be somewhat oscillatory and noisy.

Shepp and Logan modified the above filter function and proposed following;

$$\varphi(ka) = \frac{-4}{a^2(4k^2-1)}\ , \text{for } k = 0, \pm 1, \cdots \qquad (2)$$

$$\Phi(\omega) = \left|\frac{2}{a} \sin \frac{\omega a}{2}\right| \text{sinc}^2(\tfrac{\omega a}{2})$$

This filter improved the density resolution and noise characteristics in the most of X-ray tomographic scanner types.

As the CT systems are improved, the range of application is expanded and a more generalized filter is required to adapt the system noise, aliasing effects, and desired density resolutions, etc.

One generalized reconstruction filter proposed is as follows,

$$\Phi(\omega) = \frac{2}{a} \sin \frac{\omega a}{2} (p + q \cos \omega a + r \cos 2\omega a) \qquad |\omega| \leq \frac{2\pi}{a} \qquad (3)$$

In the above equation, the coefficients, p, q, and r should be chosen such that they may be applicable to a particular system of interest.

In the present paper a number of simulated results will be presented for the comparison of the various existing filters and the proposed one in this paper. Considering the importance of the filter function in 3-D reconstruction, some theoretical development of the filter will be presented.

REFERANCES

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