가변경계조건을 갖는 새로운 칼만필터 및 레귤레이터 구성

(Linear-Quadratic-Gaussian Regulators with Moving Horizons)

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While the standard linear-quadratic-Gaussian problem has fixed horizons, this paper considers the LQG problem with moving horizons. By the seperation principle the solution will be given by the Kalman filter with the approaching horizon and the LQ regulator with the receding horizon. Sufficient conditions on weighting matrices are derived under which the filter and regulator are asymptotically stable. It will be shown that the computation method of the moving-horizon LQG regulator is better than that of the standard LQG regulator. The performance measure between the two optimal controls will be compared. A simulation result is given in order to show the usefulness of the moving-horizon LQG regulator.

I. Introduction

It has been understood that linear feedback systems require following properties among others;(i) the feedback system must be stable. Feedback gains must be defined for all $t \in [t_o, \infty)$ for stability analysis.(ii) the feedback gain could be time-varying, but it must be time-invariant for time-invariant systems.(iii) computation of feedback gains must be easy.(iv) the system must have robustness with respect to perturbations.(v) the output feedback is preferable to the state feedback.

The above requirements can partially be achieved by the linear quadratic(LQ) optimal control[4],whose steady state solution satisfies (i), (ii), and (iv), and also by the linear-quadratic-Gaussian(LQG) stochastic optimal control investigated by Kalman and Bucy[5], whose steady state solution satisfies (i), (ii), (iv), and (v). The LQG theory was a breakthrough in the control history and is well known now [8]. The LQG regulator has been shown to consist of the Kalman-Bucy filter and LQ regulator. For finite-time optimal problem initial and terminal times are fixed for this standard problem. Since the steady state LQ regulator requires the terminal time $t_f = \infty$ and the steady state Kalman-Bucy filter the initial time $t_o = \infty$, the computation of the Riccati matrix equation is difficult because of integration over infinite interval.

Thomas[6] considered LQ regulators with the receding horizon with control energy and the moving terminal constraint and also linear state estimators seperately with some constraints. The stability property of this receding-horizon control was given by Kleinman[7]. Kwon and Pearson[1] considered receding-horizon LQ problems for time-varying systems with general quadratic cost and showed its optimal control satisfies (i), (ii), (iii), and (iv). This paper considers LQG regulator problems with moving horizons and will show that its optimal solution satisfies (i) through (v).

II. Main Results

Consider linear stochstic systems

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t) \qquad (1)$$

$$y(t) = C(t)x(t) + v(t) \qquad (2)$$

where the state $x(t) \in R^n$, the control $u(t) \in R^m$, the output $y(t) \in R^p$, and noises $w(t)$ and v(t) satisfy

$$E[x(t_o)] = x_o,\ E[(x(t_o)-x_o)(x(t_o)-x_o)'] = \Sigma_o$$
$$E[w(t)] = E[v(t)] = 0,\ E[w(t)v'(t)] = 0 \tag{3}$$
$$E[w(t)w'(t)] = Q_1(t),\ \ E[v(t)v'(t)] = R_1(t).$$

The problem is to find a optimal control which minimizes the moving cost

$$J(u) = E[\ x'(t+T_r)F_2(t+T_r)x(t+T_r) + \int_{t-T_f}^{t+T_r}\{ x'(t)Q_2(t)x(t) + u'(t)R_2(t)u(t)\}\,dt \tag{4}$$

at instant time t, with variance$[x(t-T_f)]=F_1(t)$, where T_r is the receding horizon ant T_f the approaching horizon for Kalman-Bucy filters. The optimal solution is derived from the standard LQG problem by replacing t_o and t_f by $t-t_f$ and $t+T_r$ respectively. The optimal filter with the approaching horizon is given by

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + \Sigma(t-T_f,t)C'(t)R_1^{-1}(t)(y(t)-C(t)\hat{x}(t)) \tag{5}$$

where

$$\frac{\partial\Sigma(\tau,t)}{\partial t} = A(t)\Sigma(\tau,t) + \Sigma(\tau,t)A'(t) - \Sigma(\tau,t)C'(t)R^{-1}(t)C(t)\Sigma(\tau,t) + Q_1(t) \tag{6}$$

and $\Sigma(\tau,\tau) = F_1(t)\tau \le t$. The LQ regulator is given by

$$u(t) = -R_2^{-1}(t)B'(t)K(t,t+T_r)\hat{x}(t) \tag{7}$$

where

$$\frac{\partial K(t,\tau)}{\partial t} = A'(t)K(t,\tau) + K(t,\tau)A(t) - K(t,\tau)B(t)R_2^{-1}(t)B'(t)K(t,\tau) + Q_2(t) \tag{8}$$

and $K(\tau,\tau) = F_2(\tau)$, $t<\tau$. We investigate conditions under which the optimal systems are stable. It is noted that F_1, Q_2, R_2, and often F_2, Q_1, and R_1 are design parameters.

Theorem 1.

(1) If$\{A(t),C(t)\}$is uniformly completely observable, B(t) is bounded, $\alpha_1 I<R(t)\le \alpha_2 I$, $0\le Q(t) \le\alpha_3 I$, and $-\dot{F}_1(t) +A(t)F_1(t)+F_1(t)A'(t)-F_1(t)C'(t)R_1^{-1}(t)C(t)F_1(t) +Q_1(t)\le 0$, then the Kalman-Bucy filter with the approaching horizon (5) is uniformly asymptotically stable.

(2) If$\{A(t),B(t)\}$ is uniformly completely controllable, C(t) is bounded,$\alpha_1 I<R_2(t) <\alpha_2 I$, $0 \le Q_2(t)<\alpha_3 I$, and $\dot{F}_2(t) + A'(t)F_1(t)+F_2(t)A(t)-F_2(t)B(t)R_2^{-1}(t)B'(t)F_2(t)+Q_2(t)\le 0$, then the LQ regulator with the receding horizon(7) is uniformly asymptotically stable.

Theorem 2.

(1) Assume that all conditions in Theorem 1 are satisfied except $F_1(t)$ and $F_2(t)$. The following Kalman-Bucy filter and LQ regulator are uniformly asymptotically stable.

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + \Gamma^{-1}(t-T_f,t)C'(t)R_1^{-1}(t)(y(t)-C(t)\hat{x}(t)) \tag{9}$$

$$\frac{\partial\Gamma(\tau,t)}{\partial t} = -A'(t)\Gamma(\tau,t)- \Gamma(\tau,t)A(t) - \Gamma(\tau,t)Q_1(t)\Gamma(\tau,t) + C'(t)R_1^{-1}(t)C(t),\Gamma(\tau,\tau)=0 \tag{10}$$

and

$$u(t) = -R_2^{-1}(t)B'(t)P^{-1}(t,t+T_r)\hat{x}(t) \tag{11}$$

$$\frac{\partial P(t,\tau)}{\partial t} = -A(t)P(t,\tau) - P(t,\tau)A'(t) - P(t,\tau)Q_2(t)P(t,\tau) + B(t)R_2^{-1}(t)B(t),P(\tau,\tau)=0, \tag{12}$$

Theorem 3.

(1) The error variance of the estimator (5) and (9) satisfies

$$\Sigma(t_o,t_1) \le E[(e(t_1)-\bar{e}(t_1))(e(t_1)-\bar{e}(t_1))']\le \Gamma^{-1}(t_1-T_f,t_1) +\Phi (t_1,t_o)(\Sigma_o-\Gamma^{-1}(t_o-T_f,t_o)) \times\Phi (t_1,t_o)$$

(2) The LQ regulator with the receding horizon (7) and (11) has the following bounds:

$$x'(t_o)K(t_o,t_1)x(t_o) \le\int_{t_o}^{t_1}\{x'(t)Q_2(t)x(t)+u'(t)R_2(t)u(t)\}dt+x'(t_1)F_2(t_1)x(t_1)$$

$$\leq \quad x'(t_o)P^{-1}(t_o,t_o+T_r)x(t_o)$$

Remarks

(1) Corresponding results for time-invariant system are possible. Especially $F_1(t) = F_2(t) = 0$ for time-invariant systems in Theorem 1.
(2) For time-invariant systems feedback gains are all constants.
(3) Computation of Riccati equations is on finite intervals and thus it is easy to compute with modern computers.
(4) The case $T_r = \infty$ and $T_f = \infty$ correspond to steady state regulator and filter respectively
(5) Memory requirement for feedback gains is smaller that that of the standard problems

III. Simulation Results

The algorithms presented in this paper are tested with a practical model. The LQG regulator with moving horizons was similar to the standard LQG regulator for this model.

IV. Conclusion.

The receding horizon concept has been successfully applied to linear discrete and continuous systems [1,2,3]. This paper provides a unified approach for the LQG regulator with moving horizons. Those stochastic controls obtained in this paper will find many applications because of great many applications of LQG problems. The results in Theorem 2 can't be applied to function space models like delayed systems since the corresponding operators do not generally have inverses. However the results in Theorem 1 could be applied to function space models, though easy ways to find $F_1(t)$ and $F_2(t)$ are in order.

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