EVOLUTION OF A MASS ACCRETING PROTOSTAR OF ONE SOLAR MASS UNDER QUASI-HYDROSTATIC EQUILIBRIUM

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ABSTRACT

The evolutionary tracks of a protostar of one solar mass under quasi-hydrostatic equilibrium are computed with mass-accretion time scales of 10^3, 10^4, 10^5 and 10^6 years, and their resulting behaviors in the H-R diagram are discussed.

It is found that there exists a critical time scale of mass accretion, which reverses the course of their evolutionary tracks. A value of the critical time scale appears to lie between 10^3 and 10^4 years. The physical cause for the presence of the critical time scale is discussed. Finally, it is proposed that star formation requires at least several 10^3 years before any star is born out of dark dense interstellar clouds.

I. INTRODUCTION

Star formation and the early stages of stellar evolution have been one of the most fundamental problems in astrophysics. It is now generally believed that the existence of life is heavily dependent upon physical conditions during and immediately after the formation of stars. The recent discovery that the molecular compounds necessary for the development of life are commonly found in dense clouds in interstellar space makes the study of star formation more important and interesting. Recent advances in infrared and microwave technology have made it possible to use tools to probe physical conditions inside the dense dark clouds, which are thought to be the birth-places of stars.

We know that there are a number of very luminous, massive stars in our Galaxy. According to the current theory of stellar evolution these luminous stars are found to be considerably younger than the age of the Galaxy, indicating their later birth with respects to the formation of the Galaxy. These massive stars are often found in stellar associations, which are a group of young stars that are gravitationally unbound (loosely bound), thus expanding away from their common center of mass. Like the stellar associations and stellar clusters, most stars appear in groups. The mass of a typical group of stars ranges from 10^2 to 10^4 solar mass, which are considered to have gone through some kind of fragmentation from a larger main dense cloud.

The detailed mechanism responsible for the cloud fragmentation is not well understood. However, it has been suggested that during the early stages of collapse, the temperature inside the cloud is nearly constant because any heat generated by the isothermal compression is likely to be radiated away. Thus, Jeans' type of dynamical instability could give rise to cloud fragmentation. As the cloud collapses, the Jeans' length decreases. Consequently, smaller clouds in the larger cloud medium now become unstable dynamically. This process will probably continue until the fragmented smaller clouds become optically thick to their own radiation so that their internal temperature and pressure built up by the trapping of the radiation could stop further fragmentation. At this time, the embryonic protostellar clouds are formed and these fragments are thought to be the predecessors of individual

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stars.

Recently, some crude dynamical calculations have been made by several workers (e.g., Larson (1), Narita et. al. (2), Appenzeller (3), and Westbrook et. al. (4)) to study the collapsing stage of evolution of the embryonic protostellar cloud, assuming a homogeneous, non-rotating spherical cloud of star-like masses for simplicity. Surprisingly, the results of the calculations disagree considerably among various investigators (e.g., Westbrook (4)). However, it is generally accepted that the nature of the collapse of the protostellar cloud is extremely non-homogeneous, quickly developing a high density core by accreting infalling material from the outer parts of the cloud. Namely, the evolution of the protostars takes place roughly on dynamical time scale during which time a significant fraction of the envelope mass is accreted. Then the stellar core follows along a quasi-hydrostatic equilibrium path, accreting more mass from the envelope, and finally it reaches the pre-main sequence stage of evolution.

One of the major differences found in the hydrodynamic models computed by various workers is found to be the time scale during which the infalling matter controls the evolution of the protostars and the time duration of mass accretion from their envelope (e.g., Westbrook (4)). According to the hydrodynamic calculations the quasi-hydrostatic phase appears to be relatively long. Accordingly, we have taken hydrostatic models of protostars with mass accretion as worthy of further investigation. In the present investigation the evolution of a mass accreting protostar of one solar mass has been considered and the evolutionary behaviors in the H-R diagram are studied. The next section is devoted to our theoretical formulation and then, the results of the calculations are discussed. Finally, a brief summary and conclusion will be given.

II. THEORETICAL FORMULATION

Suppose a protostar reaches quasi-hydrostatic equilibrium, after having gone through the hydrodynamic collapse. Since the protostar accretes mass from its surrounding gas envelope, the initial photospheric surface does not contain the entire mass of the system. The rate at which the protostar accretes the mass from the surrounding can be written in the form of

$$\frac{dM}{dt}=AM^{\nu} \qquad (1)$$

where A is a constant in time and ν is a constant to be determined either from observations or theoretical consideration. However, the rate at which the mass increases by accretion is not well determined, so that we adopted the case of $\nu=1$ for simplicity, since our interest lies in qualitative behavior of our protostars' evolution;

$$\frac{dM}{dt}=AM \qquad (2)$$

Since our model protostars are in quasi-hydrostatic equilibrium, the Virial theorem is applicable so that for a gas sphere of the polytropic index n, it becomes

$$3(\gamma-1)U+\Omega=0 \qquad (3)$$

with

$$\Omega=-\frac{3}{(5-n)}\frac{GM^2}{R} \qquad (4)$$

Where U is the internal energy, M is the total mass, R is the radius of the protostar. The polytropic index n is taken to be constant, assuming that the star evolves homologously. The total energy of the star is then given by

$$E=U+\Omega=-\frac{3\gamma-4}{(\gamma-1)}\frac{1}{(5-n)}\frac{GM^2}{R} \qquad (5)$$

where γ is the ratio of the specific heat capacity to be taken as $\gamma=5/3$ in the present consideration. The luminosity L can be obtained by differentiating Equation (5) with respect to time,

$$L=-\frac{dE}{dt}=\frac{3}{4}\frac{GM^2}{R}\left(\frac{2}{M}\frac{dM}{dt}-\frac{1}{R}\frac{dR}{dt}\right) \qquad (6)$$

which should be equal to, by definition,

$$L=4\pi R^2\sigma T_e^4 \qquad (7)$$

where T_e is the effective temperature of the protostar and σ is Stefan-Boltzmann constant.

In order to solve

$$\frac{3}{4}\frac{GM^2}{R}\left(\frac{2}{M}\frac{dM}{dt}-\frac{1}{R}\frac{dR}{dt}\right)=4\pi R^2\sigma T_e^4 \tag{8}$$

it is necessary to express T_e explicitly as a function of R and M.

Since the protostar which has reached quasi-hydrostatic equilibrium has already developed rather high density surface layers, the temperature gradient is considered to be very superadiabatic. The luminosity of such a fully convective protostar can be estimated by determining the amount of energy radiated from the photosphere. However, the thin tenuous outer layers are likely to be in radiative equilibrium, because the convective velocity cannot exceed the local speed of sound. If we assume that the photosphere is in radiative equilibrium and use the Eddington approximation, the depth from which photons can escape occurs at an optical depth $\tau=2/3$, which is given by

$$\frac{2}{3}=\int_{r_p}^{\infty}\kappa\rho\ dr \tag{9}$$

by its definition. ρ is the mass density and κ is the mass absorption coefficient which can be approximated as

$$\kappa=\kappa_o P^\alpha T^\beta \tag{10}$$

according to Hayashi et. al. (5). With the chemical composition of $X=0.75$, $Y=0.23$ and $Z=0.02$ (where X, Y and Z are the fractional mass of hydrogen, helium and metals, respectively), κ_o is found to be $\kappa_o=2.63\text{x}10^{-16}$ (C.G.S.). Following a similar analysis made by Stein (6), one finds the photospheric pressure P_{ph} at $\tau=2/3$ is given by

$$P_{ph}=\left\{\frac{2(1+\alpha)}{3}\frac{GM}{k_0\ R^2}\cdot\frac{1}{T_e^\beta}\right\}^{\frac{1}{1+\alpha}} \tag{11}$$

after substituting Equation (10) and the density ρ (obtained from the equation of hydrostatic equilibrium) given by

$$\rho=-\frac{1}{g}\frac{dP}{dr} \tag{12}$$

into Equation (9).

Another relation relating T_e to P_{ph} comes from the conditions at the boundary between the radiative region and the convective zone; namely, the radiative energy flux F_r passing through the radiative equilibrium layers must be equal to the convective energy flux F_c in the convective zone. According to Stein (6), the convective flux can be approximated to be

$$F_c=\frac{3\gamma}{8}\left(\frac{k\gamma}{m_H\mu}\right)^{\frac{1}{2}}T_e^{\frac{1}{2}}\ P_{ph} \tag{13}$$

where μ is the mean molecular weight, m_H is the mass of hydrogen and k is the Boltzmann constant ($k=1.38\text{x}10^{-16}$ C.G.S.). Equating Equation (13) to the radiative flux $F_r=\sigma T_e^4$, one obtains

$$P_{ph}=\frac{8}{3\gamma}\left(\frac{\mu\ m_H}{\gamma k}\right)^{\frac{1}{2}}\sigma T_e^{3\cdot5} \tag{14}$$

Substituting Equation (14) into Equation (11), the effective temperature T_e is given by

$$T_e=\left[\frac{2}{3}(1+\alpha)\left(\frac{3\gamma}{8}\right)^{1+\alpha}\frac{1}{\sigma\kappa_o}\frac{G}{1+\alpha}\left(\frac{\gamma K}{mH}\right)^{\frac{1+\alpha}{2}}\mu^{\frac{-(1+\alpha)}{2}}\frac{M}{R^2}\right]^{\frac{1}{3\cdot5(1+\alpha)+\beta}} \tag{15}$$

With the use of the values of $\mu=1.36$, $\kappa_0=2.63\text{x}10^{-16}$ along with $\alpha=0.735$ and $\beta=3.05$ given by Hayashi (7), Equation (15) becomes simply

$$T_e=284\times\left(\frac{M}{R^2}\right)^{0\cdot10962} \tag{16}$$

or in terms of the solar mass and radius units

$$T_e=5329\times\left(\frac{m}{r^2}\right)^{0\cdot10962} \tag{17}$$

with

$$m=M/M_0,\quad r=R/R_0, \tag{18}$$

where M_0 and R_0 are the mass and the radius of the sun. The luminosity is readily determined by substituting Equation (17) into Equation (7),

$$l=\frac{L}{L_o}=0.7209\times\left(\frac{m}{r^2}\right)^{0\cdot43848}r^2. \tag{19}$$

Substituting Equation (2) and Equation (16) into Equation (8), Equation (8) can be expressed in terms of r and m,

$$\frac{dr}{dm}=\frac{r}{m}\left(2-\frac{9.763\times10^{-16}}{A}\frac{r^{2.12304}}{m^{1.56152}}\right) \quad (20)$$

which can be solved numerically with a proper set of starting values r_o and m_o at a time t_o. The constant A is a quantity associated with the e-folding time scale of mass accretion determined from Equation (2); namely,

$$M(t)=M_o e^{-A(t_0-t)} \quad (21)$$

where t_0 is the time at which our protostar completes its mass accretion. If we define a time t_s at which the mass M_o is reduced by a factor of e, (note that time here is measured backward), the quantity A is given by

$$A=\frac{1}{T_s}=\frac{1}{t_o-t_s} \quad (22)$$

which will be treated as a free parameter in the present calculation.

The choice of the time t_o is based on the following consideration. The transition from the hydrodynamic collapsing stage to the hydrostatic phase will occur before the kinetic energy of mass motion in the protostar is converted into thermal energy. In other words, a stable star will be formed when hydrogen and helium atoms are mostly all ionized throughout the star. Consequently, one could estimate the mass-radius relation at this stage of time by equating the energy of ionization of the entire hydrogen and helium atoms of the star to half the gravitational energy. The result is found to be

$$r_o=\frac{50.3}{1-0.3X}m_o. \quad (23)$$

In the present case, it becomes with $X=0.75$

$$r_o=64.53m_o. \quad (24)$$

The computational sequence is then as follows:
(1) Assign the values of the quantity A and the mass m_o of a protostar under investigation, and determine r_o from Equation (24).
(2) Solve Equation (20) numerically by means of Runge-Kutta technique with the starting values of r_o and m_o for the next step r_1 with a proper increment of mass

$$\Delta m=m(t_1)-m_o. \quad (25)$$

(3) Determine the time t_1 from Equation (21)
(4) Calculate T_e and L/L_o at $t=t_1$ from Equation (17) and Equation (19).

III. RESULTS OF CALCULATIONS AND DISCUSSIONS

In the present investigation we have considered wide ranges of the protostellar masses and the mass accretion time scale T_s (defined in Equation (22)) as described in Table 1. The quantity m_o is the mass of a protostar under consideration at $t=t_o$ which refers to the time when the mass accretion has ceased. The particular choice of the values in the table is to cover the range of the observed masses, and therefore it is quite selective. Regarding the time scale of mass accretion, it is based on estimates obtained from observations of T Tauri stars and Herbig emission stars to ensure the range of the time scale of star formation (c.f., Strom (8)).

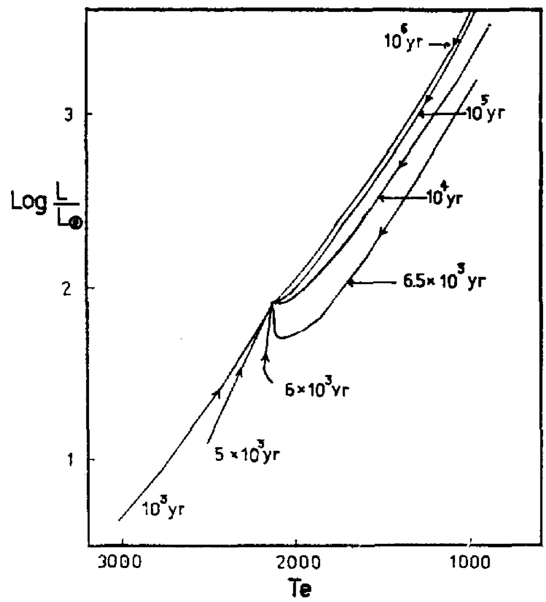

Fig. 1. Computed evolutionary tracks of one solar mass with various mass-accretion time in the H-R diagram. The arrows drawn in the figure refer to the direction of the evolution of the protostar as it accretes mass. The numerical values are the mass-accretion time under consideration.

Table 1. Values of protostellar mass m_0 at t_0 and mass accretion time scale T_s under consideration (t_0 refers to the time when the mass accression is completed)

m_o $(M/M_\odot)$	0.5 :	1 :	5 :	10 :	20 :	50
T_s	10^3 yrs:		10^4yrs:	10^5 yrs:		10^6 yrs

The computed evolutionary tracks of a protostar of one solar mass are shown in Fig. 1, where various values of mass accretion time T_s are considered. The numerical value adjacent to each solid line refers to T_s. All the evolutionary tracks have been computed backward in time, since we do not know the time when the transition from the hydrodynamic collapse to the quasihydrostatic phase occurs. However, the arrows drawn in the figures (also in Fig. 3 and Fig. 4) point to the direction of their evolutionary courses as the protostar accretes mass from their own surrounding envelope.

One of the interesting results is that there exists a critical time scale T_{cr} in mass accretion, beyond which their evolutionary courses reverse directions in the H-R diagram as seen from Fig. 1. The critical mass-accretion time T_{cr} for one solar mass is found to about $6x10^3$ years. It seems that a typical critical time scale of mass accretion lies between 10^3 to 10^4 years.

The physical reason for stars reversing their evolutionary courses (depending on the choice of the specific value of T_s) appears to be due to the rate of infalling mass onto the protostars. The rate is fast enough to create sufficient luminosity from the kinetic energy of the infalling mass without getting any gravitational contribution to the luminosity. In fact, the kinetic energy from the infalling mass becomes so large that the protostar grows in size as the mass is accreted. This is illustrated in Fig. 2.

Now, a question arises natually whether or not any protostar whose time scale of accretion T_s. is less than its own critical value of T_{cr} can exist. The answer to this question is somewhat speculative, but it is highly unlikely to see such protostar in the dense gas clouds because the excessive heating from the kinetic energy of the infalling mass cloud readily cause thermal instability, and they are likely to adjust to other stable configurations through some catastrophic processes.

It appears that star formation takes at least the order of 10^4 years in accordance with recent infrared observations. However, it should be noted that the present investigation is quite crude and simple-minded and furthermore the hydrodynamic collapsing stage has not been taken into account. It would be interesting to see how the present results would compare to those of more refined hydrodynamic calculations in the future.

IV. SUMMARY AND CONCLUSION

We have considered the evolution of protostar of one solar mass with mass accretion under quasi-hydrostatic equilibrium, and their resulting evolutionary behaviors in the H-R diagram are discussed.

One of the interesting findings from the present investigation is that there exists a critical time scale of mass accretion which reverses the course of their evolutionary tracks. The physical reason is found to be due to the rate of infalling mass onto protostars, whose rate is so fast that oversufficient luminosity is created from the kinetic energy obtained from the infalling mass. The critical time scale T_{cr} is found to be nearly independent of the protostar, and a typical value of T_{cr} appears to lie between 10^3 to 10^4 years. It may be concluded that it would take several thousand years for any star to be born out of dark dense interstellar clouds.

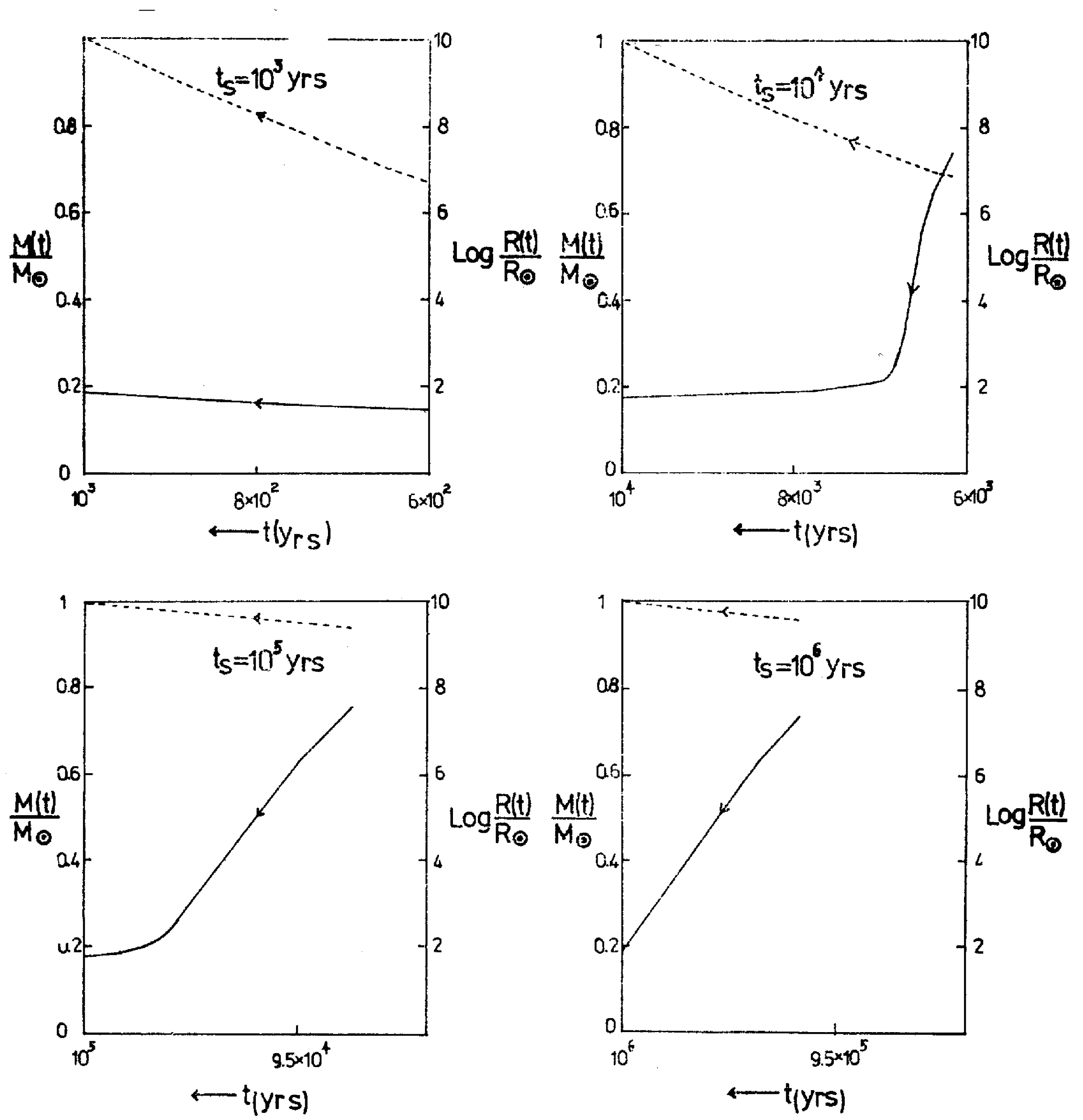

Fig. 2 Mass and radius ariations for one solar mass star during the course of its evolution. The solid lines refer to the radius and the dotted lines to the mass.

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